Mappings for Overture
A Description of the Mapping Class
and Documentation for Many Useful Mappings

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Abstract

This document describes the class `Mapping`. The Mapping class is used to define transformations. These transformations are used within Overture to define grids and stretching functions and rotations etc. The base class is called `Mapping`. Particular mappings such as a sphere or an annulus are defined by deriving a class from the base class and defining the particular transformation. A number of derived Mappings have been written including

- Various Analytical mappings: LineMapping, SquareMapping, CircleMapping, AnnulusMapping, BoxMapping, CylinderMapping, PlaneMapping, QuadraticMapping, SphereMapping
- AirfoilMapping : for creating airfoil related grids and curves (including some NACA airfoils).
- ComposeMapping : for composing two mappings
- CompositeSurface : a mapping that represents a collection of sub-surfaces.
- CrossSectionMapping : define a surface by cross-sections
- DataPointMapping : mappings defined by data points
- DepthMapping : create a 3D grid from a 2D grid by adding a variable depth.
- EllipticTransform: smooth a mapping with an elliptic transform.
- FilletMapping: create a fillet or collar grid to join two intersecting surfaces.
- HyperbolicMapping: create volume grids using hyperbolic grid generation (described else-where).
- IntersectionMapping: a mapping that is the intersection between two other mappings, such as the curve of intersection between two surfaces.
- JoinMapping: create a mapping that can join two intersecting mappings.
- MatrixMapping : define a matrix transformation by rotations, scaling, shifts etc.
- MatrixTransform : apply a matrix transformation to another mapping
- NormalMapping : define a new mapping by extending normals
- NurbsMapping : define a mapping by a NURBS, non-uniform rational b-spline.
- OffsetShell : build offset surfaces and an overlapping edge mapping to join them.
- OrthographicTransform : define an orthographic transform
- ReductionMapping : make a new Mapping from the face or edge of another mapping.
- ReparameterizationTransform : reparameterize a mapping (e.g. remove singularities, or equidistribute grid lines by arclength and curvature).
- RestrictionMapping : define a restriction to a sub-rectangle.
- RevolutionMapping : create a surface or volume of revolution
- RocketMapping : create curves related to rocket geometries.
- SmoothedPolygon : for polygons with smoothed corners
- SplineMapping: define a cubic spline curve.

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- StretchMapping : one-dimensional stretching transformations
- StretchedSquare : stretch grid lines on the unit interval.
- StretchTransform : stretch grid lines along the coordinate directions
- SweepMapping : Sweep a 2D Mapping along a curve in 3D.
- TFIMapping : define a grid from given boundary curves by transfinite-interpolation (Coon's patch).
- TrimmedMapping : define a trimmed surface in 3D, the surface has portions removed from it ("trimmed").
- UnstructuredMapping : create an unstructured representation for an existing mapping or read in an manipulate and unstructured mesh.

All these classes are described in this document.

Contents

1 Introduction

The C++ class "Mapping" can be used to define the "mappings" (transformations) and their properties. For example, each component grid in an overlapping grid will contain a member function that defines the mapping from the unit square (or unit cube) onto the domain covered by the grid. This mapping may in turn be defined in terms of the curves (or surfaces) that form its boundaries. Stretching functions as well as rotations and scalings are all defined by mappings.

New mappings are defined by derivation from the base class "Mapping". For example, the class "MatrixMapping" is a derived class that defines transformations such as rotations, scalings and translations.

The mapping class can be used to define mapping functions for curves, areas, surfaces, volumes etc.:

$$f : \mathbf{R}^{domainDimension} \rightarrow \mathbf{R}^{rangeDimension} \quad domainDimension \leq rangeDimension \ , \ rangeDimension = 0,1,2,3$$

For example a curve in 2D would have $(domainDimension, rangeDimension) = (1,2)$ and a volume in 3D would have $(domainDimension, rangeDimension) = (3,3)$

$\mathbf{R}^{domainDimension}$ is called the **domain** of the mapping while $\mathbf{R}^{rangeDimension}$ is called the **range**.

The domain will either be **parameter space** (i.e. unit line, unit square, or unit cube) or **cartesian space** (i.e. physical space with coordinates (x_1, x_2, x_3)). Similarly, the range is either parameter space or cartesian space.

Figure 1: Class diagram for a Mapping

1.1 Example:

Here is a simple example of creating and evaluating a mapping and its inverse. (file `example1.C`)

```
#include "Mapping.h"
#include "BoxMapping.h"

int
main()
{
  int axis1=0;  int axis2=1;  int axis3=2;

  // -- Define a box in 3D

  BoxMapping cube(1.,2.,1.,2.,1.,2.)  ;                   // create a cube: [1,2]x[1,2]x[1,2]

  cube.setName(Mapping::mappingName,"cube");              // give the mapping a name

  cube.setIsPeriodic(axis1,Mapping::derivativePeriodic);  // periodic in x direction

  RealArray r(1,3),x(1,3),xr(1,3,3),rx(1,3,3);            // evaluate only 1 point

  r(0,axis1)=.25; r(0,axis2)=.5; r(0,axis3)=.75;
  r.display("here is r");
  cube.map( r,x,xr );                            // evaluate the mapping and derivatives: r --> (x,xr)
  x.display("here is x after map");

  r=0;
  cube.inverseMap( x,r,rx );                     // evaluate the inverse mapping: x --> (r,rx)
  r.display("here is r after inverseMap");

  return 0;
}

```

2 Class Mapping

The base class for mappings is the class `Mapping`.

2.1 Enum Types

The following enum types are members of Class Mapping.
Here are the enumerators for the possible spaces for the domain and range

```
enum mappingSpace{
                   parameterSpace,    // bounds are [0,1]
                   cartesianSpace }   // default (-infinity,infinity)
                 };
```

For example, a stretching function will normally map from `parameterSpace` to `parameterSpace`. A square grid will usually be a mapping from `parameterSpace` to `cartesianSpace` (i.e. physical space with coordinates (x_1, x_2, x_3)). A rotation will normally be a mapping from `cartesian space` to `cartesian space`.

Here are the enumerators used to define the periodicity of the mapping (i.e. possible values for `getIsPeriodic`)

```
enum periodicType
{
  notPeriodic,
  derivativePeriodic,    // Derivative is periodic but not the function
  functionPeriodic       // Function is periodic
};
```

Here are the enumerators for the coordinate systems that we can use for the domain or the range

```
enum coordinateSystem{
                       cartesian,                 //  x,y,z
                       spherical,           //  phi/pi, theta/2pi, r
                       cylindrical,         //  theta/2pi, z, r
                       polar,               //  r, theta/2pi, z
                       toroidal             //  theta1/2pi, theta2/2pi, theta3/2pi
                     };
```

Coordinate systems are discussed in greater detail later.

Here are the enumerators for the items that we save names for in the form of character strings,

```
enum mappingItemName
{
  mappingName,      // mapping name
  domainName,       // domain name
  rangeName,
  domainAxis1Name, // names for coordinate axes in domain
  domainAxis2Name,
  domainAxis3Name,
  rangeAxis1Name,  // names for coordinate axes in range
  rangeAxis2Name,
  rangeAxis3Name
};
```

The names are assigned and retrieved with the the member functions `setName` and `getName`.
Here are the enumerators used to supply options to `setBasicInverseOption`

```
  enum basicInverseOptions  // options for basicInverse
  {
    canDoNothing,
    canDetermineOutside,
    canInvert
  };
```

Use the `setBasicInverseOption` or `getBasicInverseOption` functions to retrieve or change these values.
Here are enumerators for the types of Mapping coordinate systems, these are used to optimize the computation of difference approximations to functions defined on grids derived from this mapping.

```
  enum mappingCoordinateSystem
  {
    rectangular,                // rectangular mapping
    conformal,                  // conformal              : metric tensor is diagonal and ...
    orthogonal,                 // orthogonal mapping     : metric tensor is diagonal
    general                     // general transformation : no special properties
  };
```

Use the `setMappingCoordinateSystem` and `getMappingCoordinateSystem` functions to retrieve or change the mapping coordinate system.

2.2 Member Functions

In the following `real` will denote either `float` or `double`.

2.2.1 constructor

Mapping(int domainDimension_ =3,
int rangeDimension_ =3,
mappingSpace domainSpace_ =parameterSpace,
mappingSpace rangeSpace_ =cartesianSpace,
coordinateSystem domainCoordinateSystem_ =cartesian,
coordinateSystem rangeCoordinateSystem_ =cartesian)

Description: Default Constructor.

domainDimension_ (input):

rangeDimension_ (input):

domainSpace_ (input):

rangeSpace_ (input):

domainCoordinateSystem_ (input):

rangeCoordinateSystem_ (input):

2.2.2 basicInverse

void
basicInverse(const realArray & x,
realArray & r,
realArray & rx =nullDistributedArray,
MappingParameters & params =Overture::nullMappingParameters())

Description: A derived class may optionally define this function it the class knows how to rapidly compute the inverse of the mapping (by an analytic formula for example).

2.2.3 epsilon

real
epsilon()

Description: Return the tolerance used by the Mappings.

2.2.4 secondOrderDerivative

void
secondOrderDerivative(const Index & I,
const realArray & r,
realArray & xrr,
const int axis,
const int & rAxis)

Description: compute second derivatives of the mapping by finite differences

I (input) :

r (input) : evaulate at these points, r(I,0:domainDimension-1).

xrr (output):

axis (input): compute the derivative of x(axis,I)

rAxis (input): compute the second derivative along the direction rAxis.

2.2.5 display

void
display(const aString & label) const

Description: Write the values of the Mapping parameters to standard output.

2.2.6 getIndex

Index
getIndex(const realArray & r,
realArray & x,
const realArray &xr,
int & base0,
int & bound0,
int & computeMap0,
int & computeMapDerivative0)

Description: Return an Index operator for loops in the map and inverseMap functions Also compute the members:

computeMapping : TRUE or FALSE

computeMappingDerivative : TRUE or FALSE

base : base for Index

bound : bound for the Index

NOTE: do note make x "const" so we check that this routine is called correctly from map and inverseMap

2.2.7 get

int
get(const GenericDataBase & dir, const aString & name)

Description: Get this object from a sub-directory called "name"

2.2.8 getID

int
getID() const

Description: Get the current value for the Mapping identifier, a unique number to use when saving the Mapping in a database file. This value is used to avoid having multiple copies of a Mapping saved in a data base file.

2.2.9 setID

void
setID()

Description: Set a new value for the Mapping identifier, a unique number to use when saving the Mapping in a database file. This value is used to avoid having multiple copies of a Mapping saved in a data base file.

2.2.10 getBasicInverseOption

basicInverseOptions
getBasicInverseOption() const

Description:

2.2.11 getBoundaryCondition

int
getBoundaryCondition(const int side, const int axis) const

Description: Return the boundary condition code for a side of the mapping. A positive value denotes a physical boundary, 0 an interpolation boundary and a negative value a periodic direction.

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.2.12 getBoundingBox

RealArray
getBoundingBox(const int & side = -1,
const int & axis = -1) const

Description: Return the bounding box for the Mapping (if side¡0 and axis¡0) or the bounding box for a particular side.

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.2.13 getBoundingBox

const BoundingBox &
getBoundingBoxTree(const int & side,
const int & axis) const

Description: Return the BoundingBox (tree) for a side of a Mapping.

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.2.14 getCoordinateEvaluationType

int
getCoordinateEvaluationType(const coordinateSystem type) const

Description:

2.2.15 getDomainBound

Bound
getDomainBound(const int side, const int axis) const

Description:

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.2.16 getDomainCoordinateSystem

coordinateSystem
getDomainCoordinateSystem() const

Description:

2.2.17 getDomainCoordinateSystemBound

Bound
getDomainCoordinateSystemBound(const int side, const int axis) const

Description:

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.2.18 getDomainDimension

int
getDomainDimension() const

Description:

2.2.19 getDomainSpace

mappingSpace
getDomainSpace() const

Description:

2.2.20 getGridDimensions

int
getGridDimensions(const int axis) const

Description:

axis (input): axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.2.21 getGrid

const realArray&
getGrid(MappingParameters & params = nullMappingParameters())

Description: Return an array that holds the values of this Mapping evaluated on an array of equally spaced points. Note that this array may or may not contain ghost points. If x denotes the array that is returned, then the values that are guaranteed to be there are

$$x(0:n_0,0:n_1,0:n_2,0:rangeDimension-1)$$

where $n_i = getGridDimensions(i) - 1$. Thus the valid values will always start with base 0 in the array. The array x may have ghost points in which case the base will be less than 0 and the bound greater than n_i.

Return value: An array x

Note: For efficiency the array is returned by reference. Thus **you should not alter the array that is returned by this routine**.

2.2.22 getInvertible

int
getInvertible() const

Description:

2.2.23 getIsPeriodic

periodicType
getIsPeriodic(const int axis) const

Description:

axis (input): axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.2.24 getMappingCoordinateSystem

mappingCoordinateSystem
getMappingCoordinateSystem() const

Description:

2.2.25 getName

aString
getName(const mappingItemName item) const

Description: Return a name from enum mappingItemName:

- **mappingName** : mapping name
- **domainName** : domain name
- **rangeName** :
- **domainAxis1Name** : names for coordinate axes in domain
- **domainAxis2Name** :
- **domainAxis3Name** :
- **rangeAxis1Name** : names for coordinate axes in range
- **rangeAxis2Name** :
- **rangeAxis3Name** :

item (input): return the name of this item.

2.2.26 getParameter

real
getParameter(const realParameter & param) const

Description: Return the value of a parameter used by the Mapping or the ApproximateGlobalInverse or the ExactLocalInverse.

- **THEnonConvergenceValue** : value given to ”r” value of the inverse when there is no convergence. This is currently equal to 10. and cannot be changed.
- **THEnewtonToleranceFactor** : convergence tolerance is this times the machine epsilon. Default=100. ?
- **THEnewtonDivergenceValue** : newton is deemed to have diverged if the r value is this much outside [0,1]. The default value is .1 and so Newton is deemed to have diverged when the r value is outside the range [-.1,1.1]
- **THEnewtonL2Factor** : extra factor for finding the closest point to a curve or surface, default=.1. This factor allows a less strict convergence factor if the target point is far from the mapping. Decrease this value if you want a more accurate answer. You may also have to decrease this value for mappings that have poor parameterizations.
- **THEboundingBoxExtensionFactor** : relative amount to increase the bounding box each direction. The bounding box can be increased in size to allow the inverse function to still converge for nearby points. The default value is .01. ***Actually*** only the bounding boxes for the highest leaves in the bounding box tree are extended by this factor. The bounding boxes for all other nodes (and the root) are just computed from the size of the bounding boxes of the two leaves of the node.
- **THEstencilWalkBoundingBoxExtensionFactor** : The stencil walk routine that finds the closest point before inversion by Newton’s method will only find the closest point if the point lies in a box that is equal to the bounding box extended by this factor in each direction. Default =.2

2.2.27 setParameter(int)

int
getParameter(const intParameter & param) const

Description: Set the value of a parameter used by the Mapping or the ApproximateGlobalInverse or the ExactLocalInverse.

- **THEfindBestGuess** : if true, always find the closest point, even if the point to be inverted is outside the bounding box. Default value is false.

2.2.28 getPeriodVector

real
getPeriodVector(const int axis, const int direction) const

Description: For a mapping with getIsPeriodic(direction)==derivativePeriodic this routine returns the vector that determines the shift from the 'left' edge to the 'right' edge.

axis (input): axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < rangeDimension$, are the components of the vector.

direction (input) : direction =0,1,...,domainDimension

2.2.29 getRangeBound

Bound
getRangeBound(const int side, const int axis) const

Description:

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.2.30 getRangeCoordinateSystem

coordinateSystem
getRangeCoordinateSystem() const

Description:

2.2.31 getRangeCoordinateSystemBound

Bound
getRangeCoordinateSystemBound(const int side, const int axis) const

Description:

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.2.32 getRangeDimension

int
getRangeDimension() const

Description:

2.2.33 getRangeSpace

mappingSpace
getRangeSpace() const

Description:

2.2.34 getShare

int
getShare(const int side, const int axis) const

Description:

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.2.35 getShare

real
getSignForJacobian() const

Description: Return the sign of the jacobian, 1 (right handed coordinate system) or -1 (left handed). This may only make sense for some mappings.

2.2.36 getTypeOfCoordinateSingularity

coordinateSingularity
getTypeOfCoordinateSingularity(const int side, const int axis) const

Description:

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.2.37 hasACoordinateSingularity

int
hasACoordinateSingularity() const

Description: return true if the Mapping has a coordinate singularity

2.2.38 intersects

int
intersects(Mapping & map2,
const int & side1 =-1,
const int & axis1 =-1,
const int & side2 =-1,
const int & axis2 =-1,
const real & tol = 0.) const

Description: Determine one mapping (or a face of the mapping) intersects another mapping (or the face of another mapping.

map2 (input): check intersect with this Mapping.

side1,axis1 (input): Check this face of this mapping (by default check all faces).

side2,axis2 (input): Check this face of map2 (by default check all faces).

tol (input) : increase the the size of the bounding boxes by tol*(box size) when determining whether the mappings intersect. Thus choosing a value of .1 will cause the Mappings to intersect provided they are close to each other while a value of -.1 will cause the mappings to intersect only if they overlap sufficiently. Return value : TRUE if the face (side,axis) of map intersects this mapping.

2.2.39 inverseMap

void
inverseMap(const realArray & x,
realArray & r,
realArray & rx =nullDistributedArray,
MappingParameters & params =Overture::nullMappingParameters())

Description: — Here is the generic inverse —-

x (input) : invert these points. The dimensions of this array will determine which points are inverted.

r (input/output) : On input this is an initial guess. If you know a reasonable initial guess then supply it, If you don't know an initial guess then set r=-1. for those points that you do not know a guess. If you do not know a guess then do NOT specify some valid value like .5 since this will probably be slower than allowing the value to be automatically generated.

rx (output): the derivatives of the inverse mapping.

params (input) : **params.computeGlobalInverse** : TRUE means compute a full global inverse, FALSE means only compute a local inverse using the initial guess supplied in r

params.periodicityOfSpace :

params.periodVector :

2.2.40 inverseMapC

void
inverseMapC(const realArray & x,
const realArray & r,
const realArray & rx =nullDistributedArray,
MappingParameters & params =Overture::nullMappingParameters())

Description: This version of inverseMap defines x and xr to be const (even though they really aren't). It can be used for some compilers (IBM:xlC) that don't like passing views of arrays to non-const references, as in mapping.inverseMapC(r(I),x(I),xr(I))

2.2.41 inverseMapGrid

void
inverseMapGrid(const realArray & x,
realArray & r,
realArray & rx =nullDistributedArray,
MappingParameters & params =Overture::nullMappingParameters())

Description: inverseMap a grid of points.

This version of inverseMap assumes that the input array is of the form of a grid of points:

```
if rangeDimension==1 then x can be of the form
     x(a1:a2,0:d-1)
     x(a1:a2,0:0,0:d-1)
     x(a1:a2,0:0,0:0,0:d-1)
if rangeDimension==2 then x can be of the form
     x(a1:a2,b1:b2,0:d-1)
     x(a1:a2,b1:b2,0:0,0:d-1)
if rangeDimension==3 then x can be of the form
     x(a1:a2,b1:b2,c1:c2,0:d-1)
```

The output is in a similar form

x (input) : evaluate the inverse mapping at these points, where

r (input/output) : if r has enough space, then compute the inverse mapping. You must supply an initial guess. Choose r=-1. if you don't know a good guess.

rx (output) : if rx has enough space, then compute the derivatives of the inverse mapping.

params (input/output) : holds parameters for the mapping.

2.2.42 map

void
map(const realArray & r,
realArray & x,
realArray & xr =nullDistributedArray,
MappingParameters & params =Overture::nullMappingParameters())

Description: Here is the transformation that defines the mapping.

r (input): r(base:bound,0:d) - evaluate the mapping at these points, where d=domainDimension-1

x (output) : - if x has enough space, x(base:bound,0:r), then compute the mapping. Here r=rangeDimension-1. Do not compute the mapping if x is not large enough

xr (output) : - if xr has enough space, xr(base:bound,0:r,0:d), then compute the derivatives of the mapping.

params (input): - holds parameters for the mapping.

2.2.43 mapC

void
mapC(const realArray & r,
const realArray & x,
const realArray &xr =nullDistributedArray,
MappingParameters & params =Overture::nullMappingParameters())

Description: This version of map defines x and xr to be const (even though they really aren't). It can be used for some compilers (IBM:xlC) that don't like passing views of arrays to non-const references, as in mapping.mapC(r(I),x(I),xr(I))

2.2.44 mapGrid

void
mapGrid(const realArray & r,
realArray & x,
realArray & xr =nullDistributedArray,
MappingParameters & params =Overture::nullMappingParameters())

Description: Map a grid of points.

This version of map assumes that the input array is of the form of a grid of points:

```
if domainDimension==1 then r can be of the form
     r(a1:a2,0:d-1)
     r(a1:a2,0:0,0:d-1)
     r(a1:a2,0:0,0:0,0:d-1)
if domainDimension==2 then r can be of the form
     r(a1:a2,b1:b2,0:d-1)
     r(a1:a2,b1:b2,0:0,0:d-1)
if domainDimension==3 then r can be of the form
     r(a1:a2,b1:b2,c1:c2,0:d-1)
```

The output is in a similar form

r (input) : evaluate the mapping at these points, where

x (output) : if x has enough space, then compute the mapping.

xr (output) : if xr has enough space, then compute the derivatives of the mapping.

params (input/output) : holds parameters for the mapping.

2.2.45 mappingHasChanged

int
mappingHasChanged()

Access: protected

Description: Call this function when the mapping has changed

2.2.46 gridIsValid

bool
gridIsValid() const

Description: Return true if remakeGrid=false

2.2.47 setGridIsValid

void
setGridIsValid()

Description: Indicate that the grid is valid.

2.2.48 periodicShift

void
periodicShift(realArray & r, const Index & I)

Description: Shift r into the interval [0.,1] if the mapping is periodic (derivative or function)

2.3 project

int
project(realArray & x,
MappingProjectionParameters & mpParams)

Purpose: Project the points x(i,0:2) onto the Mapping. This is normally used to project points onto a curve in 2D or surface in 3D (i.e. domainDimension=rangeDimension-1, aka a hyperspace of co-dimension 1).

x (input) : project these points.

mpParams (input) : This class holds parameters used by the projection algorithm.

Notes: The inverse unit square coordinates will be held in the array mpParams.getRealArray(r). If you have a good guess for these values then you should supply this array.

If you want the derivatives you should dimension mpParams.getRealArray(xr) to be big enough and then they will be computed.

Note: If you want the normal (or tangent to a curve) you should dimension mpParams.getRealArray(normal) to be big enough. For curves (domainDimension==1) the normal is actually the tangent to the curve. Otherwise the normal will only make sense if the Mapping is a curve in 2D or a surface in 3D, i.e. domainDimension=rangeDimension-1.

2.3.1 put

int
put(GenericDataBase & dir, const aString & name) const

Description: save this object to a sub-directory called ”name”

2.3.2 reinitialize

void
reinitialize()

Description: Re-initialize a mapping that has changed (this will re-initialize the inverse)

2.3.3 setName

void
setName(const mappingItemName item, const aString & itemName)

Description: Assign a name from enum mappingItemName:

- **mappingName** : mapping name
- **domainName** : domain name
- **rangeName** :
- **domainAxis1Name** : names for coordinate axes in domain
- **domainAxis2Name** :
- **domainAxis3Name** :
- **rangeAxis1Name** : names for coordinate axes in range
- **rangeAxis2Name** :
- **rangeAxis3Name** :

item (input): assign this item.

itemName (input) : name to give the item.

2.3.4 setCoordinateEvaluationType

void
setCoordinateEvaluationType(const coordinateSystem type, const int trueOrFalse)

Description:

2.3.5 setTypeOfCoordinateSingularity

void
setTypeOfCoordinateSingularity(const int side, const int axis,
const coordinateSingularity type)

Description:

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.3.6 topologyMask

intArray &
topologyMask()

Description: Return the mask that represents a partial periodicity, such as a C-grid.

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.3.7 getTopology

topologyEnum
getTopology(const int side, const int axis) const

Description: Return the topology. This is primarily used to represent C-grids.

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.3.8 setTopology

void
setTopology(const int side, const int axis, const topologyEnum topo)

Description: Specify the topology. This is primarily used to represent C-grids.

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.3.9 setDomainDimension

void
setDomainDimension(const int domainDimension0)

Description:

2.3.10 setRangeDimension

void
setRangeDimension(const int rangeDimension0)

Description:

2.3.11 setBasicInverseOption

void
setBasicInverseOption(const basicInverseOptions option)

Description:

2.3.12 setBoundaryCondition

void
setBoundaryCondition(const int side, const int axis, const int bc0)

Description:

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.3.13 setShare

void
setShare(const int side, const int axis, const int share0)

Description:

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.3.14 getShare

void
setSignForJacobian(const real signForJac)

Description: Set the sign of the jacobian, 1 (right handed coordinate system) or -1 (left handed). This may only make sense for some mappings.

signForJac (input) : should be 1. or -1.

2.3.15 setMappingCoordinateSystem

void
setMappingCoordinateSystem(const mappingCoordinateSystem mappingCoordinateSystem1)

Description:

2.3.16 setIsPeriodic

void
setIsPeriodic(const int axis, const periodicType isPeriodic0)

Description:

axis (input): axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

Notes: This routine has some side effects. It will change the boundaryConditions to be consistent with the periodicity (if necessary).

2.3.17 setGridDimensions

void
setGridDimensions(const int axis, const int dim)

Description:

axis (input): axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.3.18 setInvertible

void
setInvertible(const int invertible0)

Description:

2.3.19 setParameter(real)

void
setParameter(const realParameter & param, const real & value)

Description: Set the value of a parameter used by the Mapping or the ApproximateGlobalInverse or the ExactLocalInverse.

THEnonConvergenceValue : value given to ”r” value of the inverse when there is no convergence. This is currently equal to 10. and cannot be changed.

THEnewtonToleranceFactor : convergence tolerance is this times the machine epsilon. Default=100. ?

THEnewtonDivergenceValue : newton is deemed to have diverged if the r value is this much outside [0,1]. The default value is .1 and so Newton is deemed to have diverged when the r value is outside the range [-.1,1.1]

THEnewtonL2Factor : extra factor for finding the closest point to a curve or surface, default=.1. This factor allows a less strict convergence factor if the target point is far from the mapping. Decrease this value if you want a more accurate answer. You may also have to decrease this value for mappings that have poor parameterizations.

THEboundingBoxExtensionFactor : relative amount to increase the bounding box each direction. The bounding box can be increased in size to allow the inverse function to still converge for nearby points. The default value is .01. ***Actually*** only the bounding boxes for the highest leaves in the bounding box tree are extended by this factor. The bounding boxes for all other nodes (and the root) are just computed from the size of the bounding boxes of the two leaves of the node.

THEstencilWalkBoundingBoxExtensionFactor : The stencil walk routine that finds the closest point before inversion by Newton's method will only find the closest point if the point lies in a box that is equal to the bounding box extended by this factor in each direction. Default =.2

2.3.20 setParameter(int)

void
setParameter(const intParameter & param, const int & value)

Description: Set the value of a parameter used by the Mapping or the ApproximateGlobalInverse or the ExactLocalInverse.

THEfindBestGuess : if true, always find the closest point, even if the point to be inverted is outside the bounding box. Default value is false.

2.3.21 setPeriodVector

void
setPeriodVector(const int axis, const int direction, const real periodVectorComponent)

For a mapping with getIsPeriodic(direction)==derivativePeriodic this routine sets the vector that determines the shift from the 'left' edge to the 'right' edge.

axis (input): axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < rangeDimension$ are the components of the vector

direction (input) : direction =0,1,...,domainDimension

2.3.22 setDomainSpace

void
setDomainSpace(const mappingSpace domainSpace0)

Description:

2.3.23 setRangeSpace

void
setRangeSpace(const mappingSpace rangeSpace0)

Description:

2.3.24 setDomainCoordinateSystem

void
setDomainCoordinateSystem(const coordinateSystem domainCoordinateSystem0)

Description:

2.3.25 setRangeCoordinateSystem

void
setRangeCoordinateSystem(const coordinateSystem rangeCoordinateSystem0)

Description:

2.3.26 setDomainBound

void
setDomainBound(const int side, const int axis, const Bound domainBound0)

Description:

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.3.27 setRangeBound

void
setRangeBound(const int side, const int axis, const Bound rangeBound0)

Description:

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.3.28 setDomainCoordinateSystemBound

void
setDomainCoordinateSystemBound(const int side,
const int axis,
const Bound domainCoordinateSystemBound0)

Description:

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.3.29 setRangeCoordinateSystemBound

void
setRangeCoordinateSystemBound(const int side,
const int axis,
const Bound rangeCoordinateSystemBound0)

Description:

side, axis (input): indicates the side of the mapping, side=(0,1) (or side=(Start,End)) and axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

2.3.30 useRobustInverse

void
useRobustInverse(const bool trueOrFalse =TRUE)

Description: Use the robust form of the inverse.

2.3.31 sizeOf

real
sizeOf(FILE *file = NULL) const

Description: Return size of this object

2.3.32 update

bool
updateWithCommand(MappingInformation &mapInfo, const aString & command)

Description: Update one of the parameters common to all Mappings. This function is usually called by the update function for the derived class.

returns : true if the command was understood, false otherwise

2.3.33 update

int
update(MappingInformation & mapInfo)

Description: Update parameters common to all Mappings. This function is usually called by the update function for the derived class.

2.3.34 interactiveUpdate

int
interactiveUpdate(GenericGraphicsInterface & gi)

Description: Update Mapping parameters. This virtual function will call the update function for the derived Mapping. Use this function if you don't need to pass other Mapping information.

gi (input) : use this graphics interface.

2.3.35 Periodic Mappings

The possible values returned by the function `getIsPeriodic` or passed to the function `setIsPeriodic` are given by the enumerator `periodicType`:

```
  enum periodicType
  {
    notPeriodic,
    derivativePeriodic,    // Derivative is periodic but not the function
    functionPeriodic       // Function is periodic
  };
```

2.4 Member function `map`

Here is an example of an implementation of the `map` member function. The mapping function is implemented so that it can evaluate the mapping for an array of points.

```
  const int axis1 = 0;
  const int axis2 = 1;

void map( realArray & r, realArray & x, realArray & xr = nullArray, MappingParams & params = nullParams)
{
    Index I = getIndex( r,x,xr,base,bound,computeMap,computeMapDerivative );

    if( computeMap )
    {
      x(I,axis1)=2.*r(I,axis1)+xa;
      x(I,axis2)=2.*r(I,axis2)+ya;
    }
    if( computeMapDerivative )
    {
      xr(I,axis1,axis1)=2.;
      xr(I,axis1,axis2)=0.;
      xr(I,axis2,axis1)=0.;
      xr(I,axis2,axis2)=2.;
    }
}
```

The function `getIndex` returns an A++ index object that can be used when evaluating the mapping. Alternatively the variables `base` and `bound` can be used; note that `I.getBase(axis1)=base` and I.getBound(axis1)=bound. `getIndex` is described later in this section.

The default argument for the `xr` array is `nullArray` which is a static member function of the Mapping Class.

The argument `params` is not used in this example. It is used, for example, to indicate whether the derivatives should be returned in a different coordinate system, such as sphericalPolar. The default argument for `params` is `nullparms` which is a static member function of the MappingParams Class.

2.5 Member function `getIndex`

The function `getIndex` returns an Index object that can be used for A++ operations. The function `getIndex` also assigns values to the variables `base`, `bound`, `computeMap` and `computeMapDerivative`. Note that `base` and `bound` are consistent with the base and bound of the Index object returned by `getIndex`. The base and bound and Index object are determined by the first dimension of `r`. For example if `r` is dimensioned `r(0:9,3)` then `base=0`, and `bound=9` and `getIndex(...)=Index(base,bound-base+1)`. The variable `computeMap` is set to `TRUE` if dimensions of the array `x` can hold the index object. The variable `computeMapDerivative` is set to `TRUE` if dimensions of the array `xr` can hold the index object.

Thus, calling map with a null array (such as `Mapping::nullArray;`) in the place of `x` (or `xr`) will cause the mapping function not to evaluate `x` (or `xr`).

2.6 Member functions `inverseMap` and `basicInverse`

These member functions evaluate the inverse of the mapping for an array of points. The derivatives of the inverse mapping can also be obtained. By default an inverse is defined for all mappings whose $\mathtt{domainDimension} \le \mathtt{rangeDimension}$. This inverse uses Newton's method to invert the mapping. If the mapping can be inverted more quickly using another method then you can supply your own inverse.

- `inverseMap` : This is the primary function to call if you want to invert the mapping. This function will call `basicInverse` if it has been supplied. If the mapping is a transformation from parameter space to cartesian space (such as a mapping that defines a grid) then one normally should NOT supply this function but instead supply the `basicInverse` function. The reason for this is that the `inverseMap` must in general be able to invert the mapping when space is periodic. However, if the mapping is a transformation from parameter space to parameter space (such as a stretching transformation) or a transformation from cartesian space to cartesian space (such as a rotation) then one can supply the `inverseMap` function.
- `basicInverse` : This routine should be supplied if the mapping can be inverted quickly and the mapping is a transformation from parameter space to cartesian space. The `basicInverse` function does not need to take into account the fact that space may be periodic. To indicate that a `basicInverse` has been supplied you should use `setBasicInverseOption(canInvert)`.

The `inverseMap` function is automatically defined for all mappings whose $\mathtt{domainDimension} \le \mathtt{rangeDimension}$. When $\mathtt{domainDimension} < \mathtt{rangeDimension}$ (for example, a curve in 2D) the inverse is defined as the closest point in the least squares sense (L_2 norm).

If the mapping is a transformation from parameter space to cartesian space and the mapping can be inverted with an analytic formula then you should write the `basicInverse` member function. Here is an example

```
   const int axis1 = 0;
   const int axis2 = 1;
//==================================================================================
// Here is the basic Inverse (this is an inverse that does not know how
//  to deal with space being periodic)
//=================================================================================
void SquareMapping::basicInverse( const realArray & x, realArray & r, realArray & rx )
{
  Index I = getIndex( x,r,rx,base,bound,computeMap,computeMapDerivative );

  if( computeMap )
  {
    r(I,axis1)=(x(I,axis1)-xa)/(xb-xa);
    r(I,axis2)=(x(I,axis2)-ya)/(yb-ya);
  }
  if( computeMapDerivative )
  {
    rx(I,axis1,axis1)=1./(xb-xa);
    rx(I,axis1,axis2)=0.;
    rx(I,axis2,axis1)=0.;
    rx(I,axis2,axis2)=1./(yb-ya);
  }
}
```

If the mapping is a transformation from parameter space to parameter space or from cartesian space to cartesian space and the mapping can be inverted easily then you should write an `inverseMap` function. Here is an example from the `MatrixMapping` class:

```
void MatrixMapping::
inverseMap( const realArray & x, realArray & r, realArray & rx, MappingParameters & params )
{
  Index I = getIndex( x,r,rx,base,bound,computeMap,computeMapDerivative );

  if( (Mapping::debug/64) % 2 ==1  )
    cout << "MatrixMapping::inverseMap - params.isNull =" << params.isNull << endl;

  if( computeMap )
    for( int i=axis1; i<domainDimension; i++ )
    {
      r(I,i)=matrixInverse(i,3);              // holds shift
      for( int j=axis1; j<rangeDimension; j++)
      {
        r(I,i)=r(I,i)+matrixInverse(i,j)*x(I,j);
      }
    }

  if( computeMapDerivative )
    for( int i=axis1; i<domainDimension; i++ )
    {
      for( int j=axis1; j<rangeDimension; j++)
      {
        rx(I,i,j)=matrixInverse(i,j);
      }
    }
```

2.6.1 Member Functions `getName` , `setName`

These functions get or set the name for any of the items defined in the enum `mappingItemName`:

```
item = mappingClassName
       mappingName
       domainName
       rangeName
       domainAxis1Name
       domainAxis2Name
       domainAxis3Name
       rangeAxis1Name
       rangeAxis2Name
       rangeAxis3Name
```

These names can be used for plotting labels, for example. For example

```
  StretchMapping stretch;                              // create a mapping
  stretch.setName( mappingName, "myStretchMapping" );  // assign the mapping name
  ...
  cout << " Mapping name = " << stretch.getName( Mapping::mappingName ) << endl;
```

2.7 Coordinate singularities

The `getTypeOfCoordinateSingularity` function can be used to determine if a given side of a mapping has a singularity. The possible types of singularities are

```
  enum coordinateSingularity
  {
    noCoordinateSingularity,   //  no coordinate singularity
    polarSingularity           //  grid lines go to a point along the side
  };
```

A `polarSingularity` means that the grids lines converge to a point. For example, the standard representation for a sphere would have a `polarSingularity` on the two sides corresponding to $\phi = 0$ and $\phi = \pi$.

Information about singularities is used by the `inverseMap`.

2.8 Coordinate systems and coordinateEvaluationType

Some mappings will have the capability to return the mapping derivatives in different forms, corresponding to different coordinate systems. Use the `setCoordinateEvaluationType` function to indicate that a mapping can return the derivatives in the specified form. These alternative forms of the derivatives can be used by a grid generator to remove coordinate singularities. Here is an example taken from the `SphereMapping` Class:

```
void SphereMapping::
map( const realArray & r, realArray & x, realArray & xr, MappingParameters & params )
{
  Index I = getIndex( r,x,xr,base,bound,computeMap,computeMapDerivative );

  int i;
  switch (params.coordinateType)
  {
  case cartesian:  // mapping returned in cartesian form

    if( computeMap )
    {
      x(I,axis1)=radius(r(I,axis3))*cos(twoPi*r(I,axis2))*sin(Pi*r(I,axis1))+x0;
      x(I,axis2)=radius(r(I,axis3))*sin(twoPi*r(I,axis2))*sin(Pi*r(I,axis1))+y0;
      x(I,axis3)=radius(r(I,axis3))*cos(Pi*r(I,axis1))+z0;
    }
    if( computeMapDerivative )
    {
      xr(I,axis1,axis1)=radius(r(I,axis3))*cos(twoPi*r(I,axis2))*Pi*cos(Pi*r(I,axis1));
        ...
    }

    break;

  case spherical: // Mapping returned in spherical form : (phi,theta,r)
                  // derivatives: ( d/d(phi), (1/sin(phi))d/d(theta), d/d(r) )

    if( computeMap )
    {
      x(I,axis1)=radius(r(I,axis3))*cos(twoPi*r(I,axis2))*sin(Pi*r(I,axis1))+x0;
      x(I,axis2)=radius(r(I,axis3))*sin(twoPi*r(I,axis2))*sin(Pi*r(I,axis1))+y0;
      x(I,axis3)=radius(r(I,axis3))*cos(Pi*r(I,axis1))+z0;
    }
    if( computeMapDerivative )
    {
      xr(I,axis1,axis1)=radius(r(I,axis3))*cos(twoPi*r(I,axis2))*Pi*cos(Pi*r(I,axis1));
        ...
    }
    break;
  default:
    cerr << "Sphere::map: ERROR not implemented for coordinateType = "
         << params.coordinateType << endl;
    exit(1);
  }
}
```

2.9 Class MappingParams

Additional parameters are passed to the `map` and `inverseMap` functions by an object of the class `MappingParams`.

2.9.1 Data Members

`int isNull`	True if parameters have not been set
`int periodicityOfSpace`	=0,1,2,3
`realArray periodicityVector`	vector(s) for periodicity
`MappingWorkSpace workSpace`	work space
`int computeGlobalInverse`	TRUE by default
`coordinateSystem coordinateType`	evaluate mapping in this coordinate system
`ApproximateGlobalInverse *approximateGlobalInverse`	pointer
`ExactLocalInverse *exactLocalInverse`	pointer

If space is periodic, then the parameters `periodicityOfSpace` and `periodicityVector` must be set in calls to `inverseMap`. Here is an example:

```
#include "Mapping.h"
#include "Square.h"

void main()
{

  realArray r1(10,2), x1(10,2), xr1(10,2,2);
  realArray r2(10,2), x2(10,2), rx2(10,2,2);

  SquareMapping square();

  MappingParameters periodicParams;
  // here is where we set the periodicity of Space, this should be consistent
  // with the periodicity of ALL mappings
  periodicParams.periodicityOfSpace=1;
  periodicParams.periodicityVector(axis1,axis1)=2.; // set vector to (2,0)
  periodicParams.periodicityVector(axis2,axis1)=0.;

  cout << "=============Periodic in Space=============" << endl;
  cout << " ---Call square map with an array of values:" << endl;
  for( i=0; i<10; i++ )
  {
    r1(i,axis1)=i/9.;
    r1(i,axis2)=i/9.;
  }
  square.map( r1,x1,xr1 );  // get x1 and xr1 at an array of points
  for( i=0; i<10; i++ )
    printf(" Square: r= (%6.3f,%6.3f) x = (%7.4f,%7.4f)\n",
      r1(i,axis1),r1(i,axis2),x1(i,axis1),x1(i,axis2));

  cout << " ---Call square inverseMap with an array of values:" << endl;
  for( i=0; i<10; i++ )
  {
    x2(i,axis1)=1.5*x1(i,axis1);
    x2(i,axis2)=x1(i,axis2);
  }
  r2=1.;  // initial guess
  square.inverseMap( x2,r2,rx2,periodicParams );
  for( i=0; i<10; i++ )
    printf(" Square: x= (%6.3f,%6.3f) r = (%7.4f,%7.4f)\n",
      x2(i,axis1),x2(i,axis2),r2(i,axis1),r2(i,axis2));

}
```

The `inverseMap` member function of the `ComposeMapping` class will use the `computeGlobalInverse` parameter.

2.10 Class ApproximateGlobalInverse

This class is used to define an inverse!approximate global inverse of a mapping. The approximate global inverse computes an approximate inverse to the mapping. This approximate inverse should be good enough so that a Newton iteration will converge.

Each mapping contains a pointer to an ApproximateGlobalInverse, called `approximateGlobalInverse`. This ApproximateGlobalInverse is used by the `inverseMap` member function.

We now describe the default implementation for the ApproximateGlobalInverse. The default approximate global inverse has a discrete grid that contains values of the mapping. The inverse finds the closest point on this grid. The number of points on the grid can be set by the Mapping member function `setGridDimensions` or the actual grid to be used can be specified with the ApproximateGlobalInverse member function `setGrid`.

2.10.1 constructor

ApproximateGlobalInverse(Mapping & map0)

Description: Build an approximate inverse to go with a given mapping.

2.10.2 setGrid

// void

//==

/ /Description: // Give a grid that can be used for global search routines // The grid is assumed to have been assigned with values of the // mapping. The grid is assumed to be always declared as a // four-dimensional A++ array, grid(axis1,axis2,axis3,rangeDimension). //

/ /grid0 (input) : use this grid.

/ /gridIndexRange : index bounds for the sides of the grids //

2.10.3 getGrid

const realArray &
getGrid() const

Description: return the grid used for the inverse

2.10.4 getParameter

real
getParameter(const realParameter & param) const

Description: Return the value of a parameter.

param (input) : One of `MappingParameters::THEboundingBoxExtensionFactor` or `MappingParameters::THEstencilWalkBoundingBoxExtensionFactor`.

2.10.5 getParameter

int
getParameter(const intParameter & param) const

Description: Return the value of a parameter.

param (input) : One of `MappingParameters::THEfindBestGuess`

2.10.6 setParameter

void
setParameter(const realParameter & param, const real & value)

Description: Set the value of a parameter.

param (input) : One of `MappingParameters::THEboundingBoxExtensionFactor` or `MappingParameters::THEstencilWalkBoundingBoxExtensionFactor`.

value (input) : value for the parameter.

2.10.7 setParameter(int)

void
setParameter(const intParameter & param, const int & value)

Description: Set the value of a parameter.

param (input) : One of `MappingParameters::THEboundingBoxExtensionFactor` or `MappingParameters::THEstencilWalkBoundingBoxExtensionFactor`.

value (input) : value for the parameter.

2.10.8 useRobustInverse

void
useRobustInverse(const bool trueOrFalse =TRUE)

Description: If TRUE use the more robust approximate inverse that will work with highly stretched grids where the closest grid point x, to a given point may be many cells away from the cell containing the point x.

2.10.9 usingRobustInverse

bool
usingRobustInverse() const

Description: Return TRUE if using the more robust approximate inverse that will work with highly stretched grids where the closest grid point x, to a given point may be many cells away from the cell containing the point x.

2.10.10 sizeOf

real
sizeOf(FILE *file = NULL) const

Description: Return size of this object

2.10.11 get

int
get(const GenericDataBase & dir, const aString & name)

Description: Get this object from a sub-directory called "name"

2.10.12 put

int
put(GenericDataBase & dir, const aString & name) const

Description: save this object to a sub-directory called "name"

2.10.13 inverse

void
inverse(const realArray & x,
realArray & r,
realArray & rx,
MappingWorkSpace & workSpace,
MappingParameters & params)

Purpose: Find an approximate inverse of the mapping; this approximate inverse should be good enough so that Newton will converge

Method: 1. If space is periodic (e.g. if the grids all live on a background square which has one or more periodic edges) then we need to worry about values of x that are outside the basic periodic region. These points may have periodic images that lie inside the periodic region. We thus add new points to the list that are the periodic images that lie inside the basic square. ***NOTE*** space periodic rarley occurs and probably hasn't been tested enough.

```
--------------------
|                  |
| x                |   X
| periodic         |   initial point to invert
| image            |
|                  |
|                  |
```

```
|                  |
-------------------
```

2. For all points to invert, find the closest point on the reference grid that goes with the mapping. This grid is usually just the grid that is used when plotting the mapping. This step is performed by the function `findNearestGridPoint`

Notes: The results produced by this routine are saved in the object workSpace.

workSpace.x0 (output) : list of points to invert with possible extra points if space is periodic.

workSpace.r0 (output) : unit square coordinates of the closest point.

workSpace.I0 (output) : Index object that demarks the active points in x0 and r0.

workSpace.index0 (output) : indirect addressing array that points back to the original r array; used when there are extra points added for periodicity in space.

workSpace.index0IsSequential (output) : if TRUE then space is periodic and the index0 indirect addressing array should be used when storing results back in the user arrays r and rx.

2.10.14 initializeBoundingBoxTrees

void
initializeBoundingBoxTrees()

Description: Assign the binary tree's of Bounding Boxes

For domainDimension==1 bounding boxes are made for the whole mapping (curve in 1,2 or 3d)

For domainDimension¿1 a binary tree is created for each side Any box that has too many grid points in it is subdivided into two

```
          boundingBoxTree[side][axis]
                    |
             +------+-------+
          child1          child2
            |               |
        +---+----+     +----+------+
     child1   child2 child1     child2
       |        |     |           |

 Each box contains:
    domainBound(2,domainDimension) : index bounds for box
    rangeBound(2,rangeDimension)   : bounds of box in physical space
```

Note that these Bounding Box trees will be automatically (recursively) deleted when the destructor is called for boundingBoxTree[2][3]

2.10.15 findNearestGridPoint

void
findNearestGridPoint(const int base1, const int bound1, realArray & x, realArray & r)

Description: Find the nearest grid point by a 'stencil walk' and possibly a global search over the boundary

For each point x(i,.), i=base1,...,bound1, find the index of the closest point on the boundary r(i,.).

1. For a 1D grid start at the initial guess and look to the left or to the right depending on whether the distance decreases to the left or right.

2. For a 2D grid first do a local search, use the index arrays to indicate which points in the square to check (not all points need be searched as they would have been done on the previous checks). If the local search ends on a boundary then do a global search of all boundary points, followed by another local search.
3. For a 3D grid proceed as in 2 but use different index arrays

2.10.16 binarySearchOverBoundary

```
void
binarySearchOverBoundary( real x[3],
                          real & minimumDistance,
                          int iv[3],
                          int side = -1,
                          int axis = -1)
```

Description: Binary Search over the boundary.

For point x find the index of the closest point on the boundary iv iv should be given a value on input, current closest point

For curves or surfaces we search the entire surface for the closest point

x (input) : point to search for.

minimumDistance (input/output) : NOTE that this ”distance” is the SQUARE of the L2 norm. On input : find a point with minimum distance less than this value. On output, if minimumDistance is less than the input value then this will be the new minimum distance and iv will hold the point on the boundary that is closest.

iv (output) : Closest boundary ONLY IF a point is found that is closer than the input value of minimumDistance.

side,axis (input) : optionally specify to only search this face.

2.10.17 binarySearchOverBoundary

```
void
robustBinarySearchOverBoundary( real x[3],
                                real & minimumDistance,
                                int iv[3],
                                int side,
                                int axis )
```

Description: Robust Binary Search over the boundary. ** use this for a thin wing or c-grid ***

Method: Search for local minima in the top two bounding boxes.

For point x find the index of the closest point on the boundary iv iv should be given a value on input, current closest point

For curves or surfaces we search the entire surface for the closest point

x (input) : point to search for.

minimumDistance (input/output) : NOTE that this ”distance” is the SQUARE of the L2 norm. On input : find a point with minimum distance less than this value. On output, if minimumDistance is less than the input value then this will be the new minimum distance and iv will hold the point on the boundary that is closest.

iv (output) : Closest boundary ONLY IF a point is found that is closer than the input value of minimumDistance.

side,axis (input) : optionally specify to only search this face.

2.10.18 findNearestCell

int
findNearestCell(real x[3],
int iv[3],
real & minimumDistance)

Description: Find the nearest grid cell by a ‘stencil walk’ . This search technique may be needed for highly stretched grids since the closest grid point to x may be many cells away.

iv (input/output) : on input, the initial guess for the closest cell. On output the nearest cell (locally, may end on a boundary).

minimumDistance (output): return 0 if x is inside the cell iv. Otherwise the minimum distance is NOT computed by this routine (for efficiency) since the algorithm does not really require it.

Return values: **0** point x is inside the cell.

1 stencil walk has reached a boundary and the point is apparently not inside the cell.

2.10.19 countCrossingsWithPolygon

void
countCrossingsWithPolygon(const realArray & x,
IntegerArray & crossings,
const int & side_ =Start,
const int & axis_ =axis1,
realArray & xCross = Overture::nullRealDistributedArray(),
const IntegerArray & mask = Overture::nullIntArray(),
const unsigned int & maskBit = UINT_MAX,
const int & maskRatio1 =1,
const int & maskRatio2 =1,
const int & maskRatio3 =1)

Description: Count the number of times that the ray starting from position xv=(x,y) and extending to y=+ infinity, crosses the polygon approximation to the curve (domainDimension¡=1) or the triangulated approximation to the face of the mapping (domainDimension==3).

x(I,0: r-1) (input): set of points to check

crossings(I) (input/ouput): number of crossings for each point. **NOTE** this function will add on to the current values in this array, thus you should set this to zero on the first call, or subsequent calls, depending on your application.

side,axis_ (input): For domainDimension¿1 these will indicate the side (domainDimension==2) or the face (domainDimension==3) to check. For domainDimension==1 these values are ignored.

xCross (input/output) : If this argument is supplied then we store each crossing point in ths array as xCross(i,0:2r,cross) where cross=0,1,...,crossings(i)-1, r=rangeDimension. We save [x,y,i1,i2] if rangeDimension==2 and [x,y,z,i1,i2,i3] if rangeDimension==3. (i1,i2,i3) denotes the lower left corner of the cell that holds the intersection.

mask (input): optional arg that is used to mask out certain parts of the boundary. If this arg is given then ALL corners of a cell must have ”mask(i1,i2,i3) & maskBit” in order that a ray crossing that cell to count as an actual crossing. In other words the valid points on the boundary are marked with ”mask(i1,i2,i3) & maskBit”.

maskBit (input) : by default the mask bit is UINT_MAX == 2^m -1 (all bits on) so that invalid points would have mask(i1,i2,i3)==0

maskRatio1, maskratio2,maskRatio3 (input) : parameters from multigrid. These are the ratios of the current grid spacing to the finest grid spacing. (assuming that the grid associated with this mapping is the finest grid!).

Return value : number of times the ray crosses the polygon. For a closed curve there will be an odd number of crossings if the point is inside the polygon and and even number of crossings if the point is outside the polygon.

NOTE: If a point lies exactly on a vertical line segment then this routine will give zero crossings for this segment (it may cross other segments in which case the crossing count may be non-zero)

2.11 Class ExactLocalInverse

This class defines an exact inverse for a mapping, given a good initial guess.

Each mapping contains a pointer to an ExactLocalInverse, called `exactLocalInverse`. This ExactLocalInverse is used by the `inverseMap` member function.

The default ExactLocalInverse uses the Newton algorithm to invert the mapping (if the mapping is invertible) or uses Newton to find the closest point (L_2-norm) between a point and a surface or curve.

3 Inverting the Mapping by Newton's Method

3.1 The case of a square Jacobian

When the `domainDimension` equals the `rangeDimension` we use a fairly standard Newton's method, with some damping if the corrections are too large. Special considerations are required if the Jacobian (The Newton matrix) is singular; this could occur at a polar singularity, for example.

3.2 The case of a non-square Jacobian

When the `domainDimension` is not equal to the `rangeDimension`, such a a curve or surface, then we must define what is meant by inverting the Mapping. This amounts to finding some 'closest' point of the Mapping.

Denote the transformation defining the Mapping by

$$\mathbf{x} = \mathbf{S}(r_1, r_2)$$

where, to be specific, we consider the case of a surface in 3D.

3.2.1 Method 1 : Least Squares

Given a point $\mathbf{x}$ not on the surface, the equation $\mathbf{x} = \mathbf{S}(\mathbf{r})$ will have no solution. We need to define a best guess for the solution. By Taylor series

$$\mathbf{x} = \mathbf{S}(\mathbf{r}^{n-1}) + \nabla_{\mathbf{r}}\mathbf{S}(\mathbf{r}^n - \mathbf{r}^{n-1}) + ...$$

Linearizing the equation (Netwon's method) gives the over-determined system

$$\begin{gathered}\nabla_{\mathbf{r}}\mathbf{S}(\mathbf{r}^n - \mathbf{r}^{n-1}) = \mathbf{x} - \mathbf{S}(\mathbf{r}^{n-1}) \\ \text{or} \quad A\Delta\mathbf{r} = \Delta\mathbf{x}\end{gathered}$$

of 3 equations for the two unknowns in $\Delta\mathbf{r}$. We can 'solve' this over-determined system by least squares

$$A^T A\Delta\mathbf{r} = A^T\Delta\mathbf{x}$$

or equivalently using the QR algorithm

$$R\mathbf{r} = Q^T\Delta\mathbf{x}$$

to obtain the new guess $\mathbf{r}^n$. On convergence the residual $\Delta\mathbf{x}$ will be orthogonal to the tangent vectors on the surface, $A^T\Delta\mathbf{x} = 0$, and thus the residual will be in the direction of the surface normal.

Aside: In the hyperbolic grid generation context there is another way to define the inverse. The problem is to find a point $\mathbf{x}$ that is a given distance, d, from a point $\mathbf{x}^0$ and lying on some plane $\mathbf{n}\cdot(\mathbf{x}-\mathbf{x}^0) = 0$. In this case we have a system of three equations for three unknowns,

$$\begin{aligned}\mathbf{x} &= \mathbf{S}(\mathbf{r}) \\ \mathbf{x} &= \mathbf{x}^0 + d(\mathbf{t}_1\cos(\theta) + \mathbf{t}_2\sin(\theta))\end{aligned}$$

Here $\mathbf{t}_m$ are unit orthgonal tangent vectors on the plane and θ is the extra unknown. This system may be faster to solve than the least squares approach (?)

3.2.2 Old way: minimize l_2 distance

Minimize the l_2 distance (squared) between the point and the surface,

$$\min_{\mathbf{r}} g(\mathbf{r}) \quad \text{where} \;\; g = \|\mathbf{x} - \mathbf{S}(\mathbf{r})\|^2 = (\mathbf{x} - \mathbf{S})^T(\mathbf{x} - \mathbf{S})$$

To do this we solve $\nabla_{\mathbf{r}} g = 0$ (which could also find the maximum distance),

$$\mathbf{h}(\mathbf{r}) = \nabla_{\mathbf{r}} g = -2\nabla_{\mathbf{r}}\mathbf{S}^T(\mathbf{x} - \mathbf{S}) = 0$$

i.e.

$$\sum_k \partial_{r_i} S_k(x_k - S_k) = 0 \quad \text{for} \;\; i = 0, 1$$

This equation $\mathbf{h}(\mathbf{r}) = 0$ is solved by Newton's method,

$$\nabla_{\mathbf{r}}\mathbf{h}(\mathbf{r}^n - \mathbf{r}^{n-1}) = -\mathbf{h}(\mathbf{r}^{n-1})$$
$$\nabla_{\mathbf{r}}\mathbf{h} = H(\mathbf{x} - \mathbf{S}) - \|\nabla_{\mathbf{r}}\mathbf{S}\|^2$$

$$H_{ij} = \sum_k \partial_{r_i}\partial_{r_j} S_k(x_k - S_k) - (\partial_i S_k)^2$$

One disadvantage of this approach is that it requires the second derivative of the Mapping.

3.2.3 constructor

ExactLocalInverse(Mapping & map0)

Description: Build an ExactLocalInverse from a Mapping.

3.2.4 getParameter

real
getParameter(const realParameter & param) const

Description: Return the value of a parameter.

param (input) : one of `THEnonConvergenceValue`, `THEnewtonToleranceFactor`, or `THEnewtonDivergenceValue` or `newtonL2Factor` from the enum `MappingParameters`.

3.2.5 setParameter

void
setParameter(const realParameter & param, const real & value)

Description: Set the vaule of a parameter.

param (input) : one of `THEnonConvergenceValue`, `THEnewtonToleranceFactor`, or `THEnewtonDivergenceValue` or `THEnewtonL2Factor` from the enum `MappingParameters`.

value (input) : value to assign.

3.2.6 sizeOf

real
sizeOf(FILE *file = NULL) const

Description: Return size of this object

3.2.7 reinitialize

void
reinitialize()

Description: This will mark ExactLocalInverse as being in need of initialization. The actual call to initialize will occur when the inverse is actually used.

3.2.8 get

int
get(const GenericDataBase & dir, const aString & name)

Description: Get this object from a sub-directory called "name"

3.2.9 put

int
put(GenericDataBase & dir, const aString & name) const

Description: save this object to a sub-directory called "name"

3.2.10 initialize

void
initialize()

Description: Initialize.

3.2.11 compressConvergedPoints

```
int
compressConvergedPoints(Index & I,
                        realArray & x,
                        realArray & r,
                        realArray & ry,
                        realArray & det,
                        intArray & status,
                        const realArray & x1,
                        realArray & r1,
                        realArray & rx1,
                        MappingWorkSpace & workSpace,
                        const int computeGlobalInverse )
```

Description: Remove points that have converged or diverged so that we will only iterate on the smaller number of points that haven't converged,

3.2.12 inverse

```
void
inverse(const realArray & x1,
        realArray & r1,
        realArray & rx1,
        MappingWorkSpace & workSpace,
        const int computeGlobalInverse )
```

Description: Compute the inverse of the mapping using Newton's method. The initial guess must be good enough for Newton to converge

x1,r1,rx1 (input/output) :

workSpace (input) :

computeGlobalInverse (input): TRUE means that the approximateGlobal inverse routine was called previous to this call. In this case we look for information in the workSpace. FALSE means that the approximateGlobalInverse was not called before this call.

3.3 Registering Mappings and Reading Generic Mappings from the DataBase

In this section we describe how a mapping can be read from a database file and contructed even when the function constructing the mapping does not know the (derived) class to which the mapping belongs. For example, this situtaion occurs when a container class holds a pointer to a Mapping. The pointer is of type `Mapping*` but the pointer may point to a derived class such as `SquareMapping`. Suppose the container class is saved to a database file with the function `Container::put`. When it is read back in again with `Container::get` the `get` function will not know how to "get" the mapping.

To solve this problem each mapping class has a member function `make` (a virtual member function of the base class) that look likes

```
Mapping *SquareMapping::make( const String & mappingClassName )
{ // Make a new mapping if the mappingClassName is the name of this Class
  Mapping *retval=0;
  if( mappingClassName==className )
    retval = new SquareMapping();
  return retval;
}
```

The function `make` creates a new mapping of it's own class provided that the String passed to `make` is the name of it's class.

The Mapping Class contains a static member that is a list of pointers to Mappings, `mappingList`. Each member of the list points to an instance of a different derived mapping Class. All possible Mapping Class's that may be read from the database should have a member in `mappingList`. Here, for example, is how to add members to the `mappingList`:

```
  CircleMapping circle;
  StretchMapping stretch;
  ...
  Mapping::mappingList.add( &circle );
  Mapping::mappingList.add( &stretch );
```

The `makeMapping` member function of the Mapping Class can be used to make a Mapping corresponding to a given class name. The `makeMapping` function takes as input the name of a class that it should try to make. For example, the argument to `makeMapping` may be the String `className=="SquareMapping"`. The `makeMapping` function goes through it's list of mappings, calling the `make` member function of each mapping, until it finds the mapping class that is able to make a `"SquareMapping"`.

```
//==================================================================
//  Get a mapping from the database
//    This routine looks through the list of mapping Class's
//    that have been placed on the mappingList and tries to
//    find one that knows how to make a mapping whose name
//    is equal to the input argument className
//
//  Input:
//    const String & className
//       : name of the mapping class to get from the database file
//    Dir dir
//       : directory on the database where the mapping is stored
//    const String & name
//       : the database name for the mapping
//
//==================================================================
Mapping* Mapping::makeMapping( const String & className )
{ //  Try to construct a mapping
    Mapping *retval = 0;
    for( Item *ptr=mappingList.start; ptr; ptr=ptr->next )
      if( retval = ptr->val->make( className ) ) break;
    return retval;
}
```

Here is the definition of a container class that calls `makeMapping` in order to construct a mapping. The container class has a pointer to a generic Mapping. There is no trouble saving the mapping to a database with the `put` member function. However when it reads the mapping back from the database it must be able to construct an instance of the appropriate derived class; this is done by `makeMapping`. **Note** that we assume that each Mapping class has a data member `String className` that holds the name of the class.

```
class Container
{
public:
  Mapping *mapPointer;

  ...

  void get( const Dir & dir, const String & name )
   {
     // Make a new directory unless name="."
     Dir subDir = name=="." ? dir : dir.findDir(name);

     // Look for the className of the Mapping:
     Dir mappingDir = subDir.findDir("containedMapping");
     String mappingClassName;
     mappingDir.get( mappingClassName,"className" );

     // Make an instance of the appropriate derived Mapping class
     mapPointer = Mapping::makeMapping( mappingClassName );
     getMap=TRUE;
     mapPointer->get( subDir,"containedMapping" );   // get the mapping

   }
  void put( const Dir & dir, const String & name )
   {
     // destory the directory if it exists
     if( !dir.locateDir(name).isNull() )
       dir.destroy(name, " R");
     Dir subDir = name=="." ? dir : dir.createDir(name);

     mapPointer->put( subDir,"containedMapping" );  // save the mapping

   }

};
```

4 AnnulusMapping

This Mapping defines an annulus in two or three space dimensions

$$\begin{aligned} \theta &= 2\pi(\theta_0 + r_1(\theta_1 - \theta_0)) \\ \mathbf{x}(r_1, r_2) &= (R_0 + r_2(R_1 - R_0))(\cos(\theta) + x_0, \sin(\theta) + y_0, z_0) \end{aligned}$$

By default the annulus is parameterized with a left-handed coordinate system. You can make the system right handed by choosing the `outerRadius` to be less than the `innerRadius`.

By default the annulus is two dimensional. To make a three dimensional annulus use the `setRangeDimension()` function or use the `setOrigin(x0,y0,z0)` function with a non-zero value of `z0`.

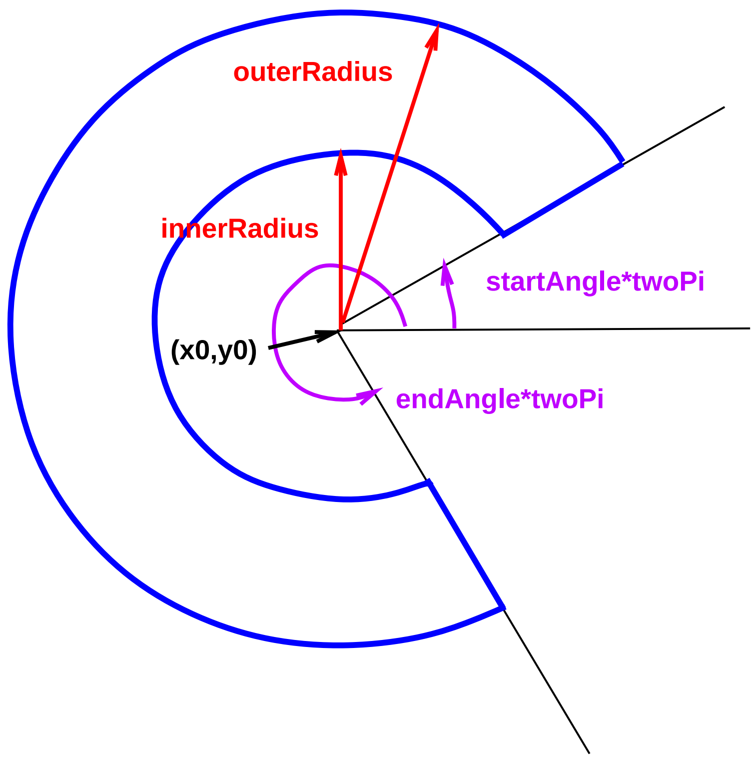


Figure 2: The AnnulusMapping defines an annulus

4.1 Constructor

AnnulusMapping(const real innerRadius_ =.5,
const real outerRadius_ =1.,
const real x0_ =0.,
const real y0_ =0.,
const real startAngle_ =0.,
const real endAngle_ =1.)

Purpose: Create an annulus.

innerRadius,outerRadius (input): inner and outer radii.

x0,y0 (input): centre for the annulus.

startAngle, endAngle (input): The initial and final ”angle” (in the range [0,1]).

4.2 setRadii

int
setRadii(const real & innerRadius_ =.5,
const real & outerRadius_ =1.)

Purpose: Define the radii of the annulus.

innerRadius,outerRadius (input): inner and outer radii of the annulus. There is NO restriction that `innerRadius` < `outerRadius`.

4.3 setOrigin

int
setOrigin(const real & x0_ =0.,
const real & y0_ =0.,
const real & z0_ =0.)

Purpose: Set the centre of the annulus. Choosing a non-zero value for `z0` will cause the `rangeDimension` of the Mapping to become 3.

x0,y0,z0 (input): centre of the annulus.

4.4 setAngleBounds

int
setAngleBounds(const real & startAngle_ =0.,
const real & endAngle_ =1.)

Purpose: Set the angular bounds on the annulus.

startAngle, endAngle (input): The initial and final ”angle” (in the range [0,1]).

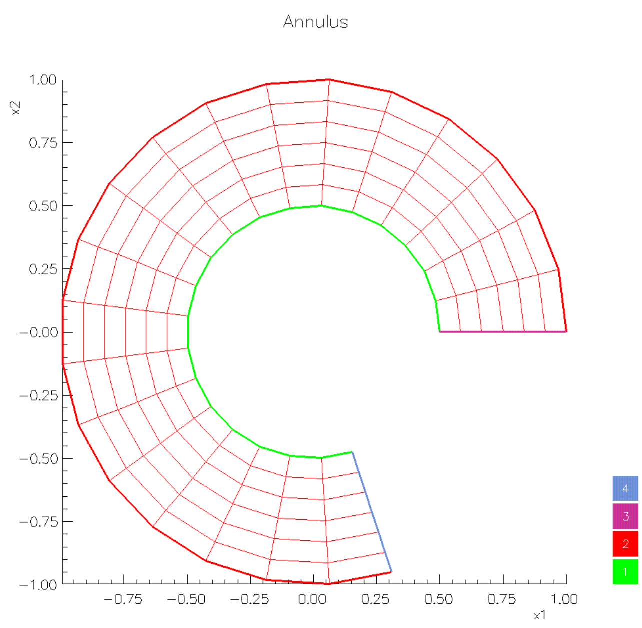


Figure 3: A mapping for a partial annulus.

5 AirfoilMapping: create some airfoil related grids or curves

5.1 NACA airfoils

The NACA 4 digit series airfoils (such as the NACA0012) are defined by

$$\begin{aligned} x_u(r) &= (r - y_t(r)\sin(\theta))c && \text{upper surface} \\ y_u(r) &= (y_c(r) + y_t(r)\cos(\theta))c && \text{upper surface} \\ x_l(r) &= (r + y_t(r)\sin(\theta))c && \text{lower surface} \\ y_l(r) &= (y_c(r) - y_t(r)\cos(\theta))c && \text{lower surface} \end{aligned}$$

where c is the chord length. The camber line, y_c, is defined by

$$\begin{aligned} y_c(r) &= c_{\max}\frac{1}{x_1^2}(2x_1 r - r^2) \qquad \text{for } 0 \le r \le x_1 \\ y_c(r) &= c_{\max}\frac{1}{(1-x_1)^2}((1-2x_1) + 2x_1 r - r^2) \qquad \text{for } x_1 \le r \le 1 \\ x_1 &= \text{ position of the maximum camber} \end{aligned}$$

and the thickness is defined by

$$y_t(r) = 5\delta(0.29690\sqrt{r} - 0.12600r - 0.35160r^2 + .28430r^3 - 0.10150r^4)$$

where

$$\delta = \text{thickness/chord}$$

The NACA[c][p][tc] airfoil is defined by:

c maximum camber/chord $\times 100$ ($c_{\max} \times 100$).

p position of maximum camber/chord $\times 10$ ($x_1 \times 10$).

tc thickness/Chord $\times 100$ ($\delta \times 100$).

Thus the NACA0012 has $c_{\max} = 0$, $x_1 = 0$ and $\delta = .12$.

5.2 Joukowsky Airfoil

The Joukowsky airfoil is defined by

$$\begin{aligned} z = x + iy &= w + \frac{1}{w} \qquad \text{z and w are complex numbers} \\ w &= ae^{i\theta} + ide^{i\delta} \\ \theta &= 2\pi r_0 \end{aligned}$$

The parameters a, d, δ in the definition have the following approximate properties,

a : $<= 1$, closer to 1 implies sharper trailing edge.

d : bigger d implies larger camber.

δ : bigger δ implies greater asymmetry between leading and trailing edges.

5.3 Member function descriptions

5.3.1 Constructor

AirfoilMapping(const AirfoilTypes & airfoilType_,
const real xa = -1.5,
const real xb = 1.5,
const real ya = 0.,
const real yb = 2.)

Description: Create a mapping for an airfoil.

Notes: An airfoil mapping can be made from oneof the following (enum AirfoilTypes)

- **arc** : grid with a bump on the bottom that is an arc of a circle.
- **sinusoid** : grid with a bump on the bottom that is an sinusoid.
- **diamond** : grid with a bump on the bottom that is a diamond.
- **naca** : a curve that is one of the NACA 4 digit airfoils.
- **joukowsky** : a curve defining a Joukowsky airfoil.

airfoilType_ (input): an airfoil type from the above choices.

xa,xb,ya,yb (input) : boundaries of the bounding box (not used for naca airfoils).

5.3.2 setBoxBounds

```
int
setBoxBounds(const real xa =-1.5,
             const real xb =1.5,
             const real ya =0.,
             const real yb =2.)
```

Description: set bounds on the rectangle that the airfoil sits in

xa,xb,ya,yb (input) : boundaries of the bounding box (not used for naca airfoils).

5.3.3 setParameters

```
int
setParameters(const AirfoilTypes & airfoilType_,
              const real & chord_ =1.,
              const real & thicknessToChordRatio_ =.1,
              const real & maximumCamber_ =0.,
              const real & positionOfMaximumCamber_ =0.,
              const real & trailingEdgeEpsilon_ =.02)
```

Description: Create a mapping for an airfoil.

Notes: An airfoil mapping can be made from oneof the following (enum AirfoilTypes)

- **arc** : grid with a bump on the bottom that is an arc of a circle.
- **sinusoid** : grid with a bump on the bottom that is an sinusoid.
- **diamond** : grid with a bump on the bottom that is a diamond.
- **naca** : a curve that is one of the NACA 4 digit airfoils.
- **joukowsky** : Joukowsky airfoil. The other parameters in the argument list do not apply in this case. Use the `setJoukowskyParameters` function instead.

airfoilType_ (input): an airfoil type from the above choices.

chord_ (input): length of the chord.

thicknessToChordRatio_ (input): thickness to chord ratio.

maximumCamber_ (input): maximum camber

positionOfMaximumCamber_ (input): position of maximum camber

trailingEdgeEpsilon_ (input) : parameter for rounding the trailing edge.

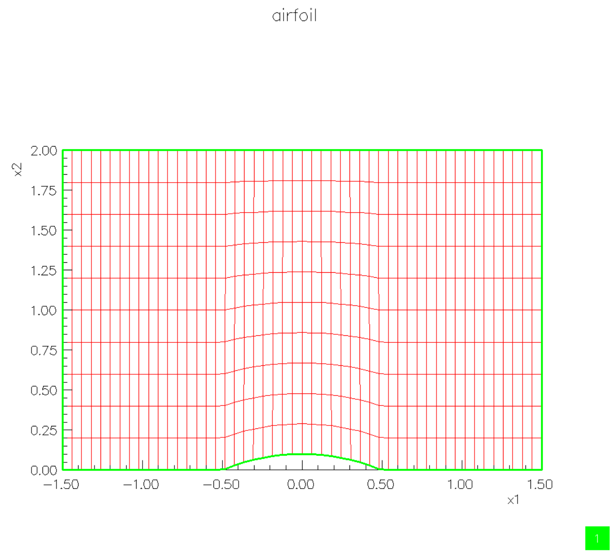
5.3.4 setJoukowskyParameters

int
setJoukowskyParameters(const real & a, const real & d, const real & delta)

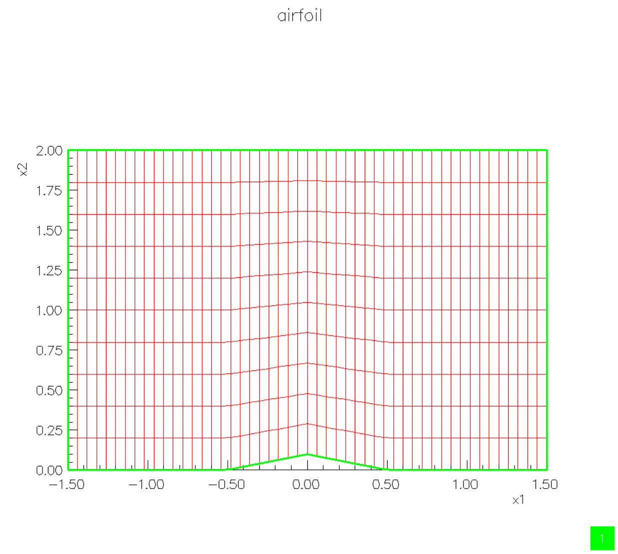
Description: Set parameters for the Joukowsky airfoil.

a,d,delta : see the documentation for a desciption of these.

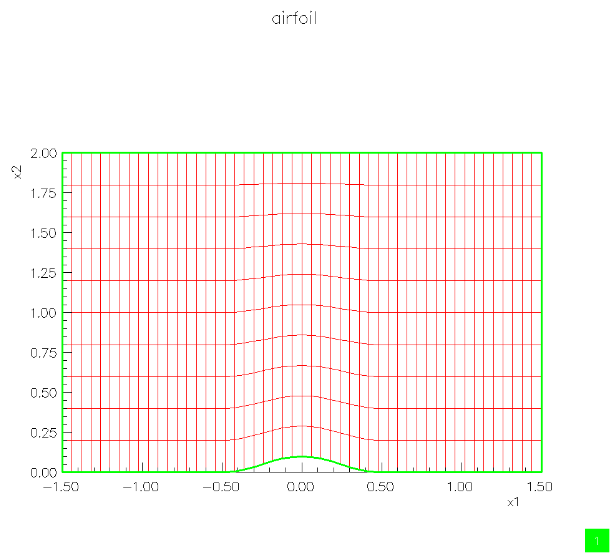
5.4 Examples



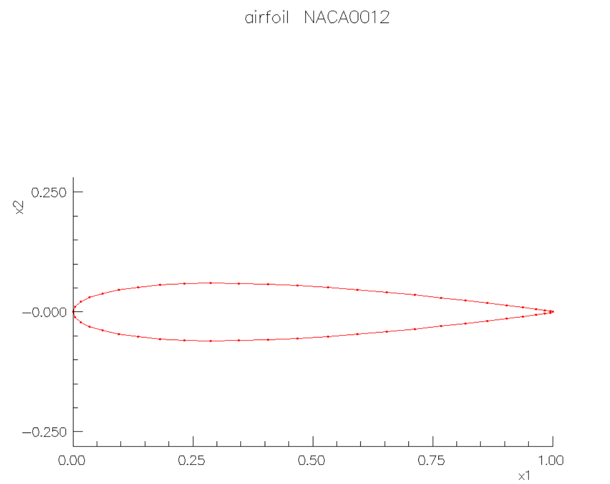
Airfoil grid created with `airfoilType=arc`



Airfoil grid created with `airfoilType=diamond`



Airfoil grid created with `airfoilType=sinusoid`



NACA0012 airfoil created with `airfoilType=naca`

6 BoxMapping

This Mapping defines a box in three-dimensions:

$$\mathbf{x}(r_1, r_2, r_3) = (x_a + r_1(x_b - x_a), y_a + r_2(y_b - y_a), z_a + r_3(z_b - z_a))$$

The box can also be rotated around any one of the coordinate directions.

6.1 Member functions

6.2 constructor

BoxMapping(
const real xMin, const real xMax,
const real yMin, const real yMax,
const real zMin, const real zMax)

Description: Build a rectangular box in 3D. The box can also be rotated around one the coordinate directions.

xMin,xMax : minimum and maximum values for x(xAxis)

yMin,yMax : minimum and maximum values for y(yAxis)

zMin,zMax : minimum and maximum values for z(zAxis)

6.3 rotate

int
rotate(const real angle,
const int axisOfRotation = 2,
const real x0 = 0.,
const real y0 = 0.,
const real z0 = 0.)

Description: Rotate the box around a coordinate direction.

angle (input) : angle in degrees.

xisOfRotation (input) : 0,1 or 2.

x0,y0,z0 (input) : rotate about this point.

6.4 getVertices

int
getVertices(real & xMin, real & xMax, real & yMin, real & yMax, real & zMin, real & zMax) const

Purpose: Return the bounds on the box.

xMin, xMax, yMin, yMax, zMin, zMax (output) : bounds on the box.

6.5 setVertices

int
setVertices(const real & xMin =0.,
const real & xMax =1.,
const real & yMin =0.,
const real & yMax =1.,
const real & zMin =0.,
const real & zMax =1.)

Purpose: Set the bounds on the box.

xMin, xMax, yMin, yMax, zMin, zMax (input) : bounds on the box.

7 CircleMapping (ellipse too)

This mapping defines a circle or ellipse in two or three dimensions:

$$\begin{aligned}\mathbf{x}(r) &= (a\cos(2\pi r)+x_0, b\sin(2\pi r)+y_0)\\ \mathbf{x}(r) &= (a\cos(2\pi r)+x_0, b\sin(2\pi r)+y_0, z_0)\end{aligned}$$

on a constant $z-plane$. A partial arc can also be defined (see the figure with AnnulusMapping).

7.1 Constructor(2D)

CircleMapping(const real & x_ =0.,
const real & y_ =0.,
const real & a_ =1.,
const real & b_ =a_,
const real & startTheta_ =0.,
const real & endTheta_ =1.)

Description: Define a circle or ellipse (or an arc there-of) in 2D, semi-axes a, and b, angle from startTheta*twoPi to endTheta*twoPi

```
x(I,axis1)=a*cos(thetaFactor*(r(I,axis1)-startTheta))+xa;
x(I,axis2)=b*sin(thetaFactor*(r(I,axis1)-startTheta))+ya;
```

x_ (input) : x coordinate of center

y_ (input) : y coordinate of center

a_ (input) : length of semi axis along x (radius for a circle)

b_ (input) : length of semi axis along y (radius for a circle)

startTheta_ (input): starting angle (in units of radians/(2 pi))

endTheta_ (input): ending angle (in units of radians/(2 pi))

7.2 Constructor(3D)

CircleMapping(const real & x_,
const real & y_,
const real & z_,
const real & a_,
const real & b_,
const real & startTheta_,
const real & endTheta_)

Description: Define a circle or ellipse (or an arc there-of) in 3D (constant z), semi-axes a, and b, angle from startTheta*twoPi to endTheta*twoPi

```
x(I,axis1)=a*cos(thetaFactor*(r(I,axis1)-startTheta))+xa;
x(I,axis2)=b*sin(thetaFactor*(r(I,axis1)-startTheta))+ya;
x(I,axis3)=za;
```

x_ (input) : x coordinate of center

y_ (input) : y coordinate of center

z_ (input) : z coordinate of center

a_ (input) : length of semi axis along x (radius for a circle)

b_ (input) : length of semi axis along y (radius for a circle)

startTheta_ (input): starting angle (in units of radians/(2 pi))

endTheta_ (input): ending angle (in units of radians/(2 pi))

8 ComposeMapping: compose two mappings

This mapping can be used to create a new mapping by composing two existing mappings.

8.1 Constructors

`Mapping()`	Default constructor
`Mapping( Mapping & mapa, Mapping & mapb)`	create a mapping, `mapb` ∘ `mapa`

8.2 Member Functions

`void map( realArray & r, realArray & x, realArray & xr )`	evaluate the mapping and derivative
`void inverseMap( realArray & x, realArray & r, realArray & rx )`	evaluate the inverse mapping and derivative
`void get( const Dir & dir, const String & name)`	get from a database file
`void put( const Dir & dir, const String & name)`	put to a database file

Here is an example of the use of the `ComposeMapping` class. The composed mapping consists of a mapping for a cube followed by a rotation mapping.

```
#include "maputil.h"

void main()
{
  BoxMapping box(0.,.5,0.,.5,0.,.5)  ;           // Define grid to be a cube

  MatrixMapping rotation  ;                      // Define a matrix mapping
  rotation.rotate( zAxis, Pi/2. );               // rotate about z axis

  ComposeMapping rotatedBox( box,rotation );     // define a mapping by composition

  r(axis1)=.5; r(axis2)=.5; r(axis3)=.5;
  rotatedBox.map( r,x,xr );                      // evaluate the mapping
}
```

9 CompositeSurface: define a surface formed from many sub-surfaces

******* This class is still under development ********

The `CompositeSurface` Mapping is used to represent a surface that is formed from a collection of sub-surfaces. This Mapping is not a normal Mapping since it does not represent a transformation from the unit square.

As an example, a CompositeSurface is used to represent the collection of NURBS and trimmed-NURBS surfaces that can be created by CAD packages. A single CompositeSurface can hold any number of these surfaces. Each sub-surface in a CompositeSurface is just any Mapping. Usually every sub-surface will actually be a surface in 3D but this is not necessary.

The most common use for a CompositeSurface is in combination with the Hyperbolic surface grid generator. This surface grid generator can grow a grid over a portion of a CompositeSurface, starting from some intial curve on the surface.

9.1 Projection onto the composite surface

The CompositeSurface has a function `project` that can be used to take one or more points in space, $\mathbf{x}_i$, and project these points onto the CompositeSurface, giving new points $\mathbf{x}_i^p$.

The hyperbolic surface grid generator, for example, will march a line of points over the CompositeSurface. At each step in it's marching algorithm, new positions will be predicted for the next position for the line of points. These predicted values are then projected exactly onto the CompositeSurface.

The projection algorithm make use of the following variables:

$\mathbf{x}$: point near the surface that needs to be projected.

s_0 : initial guess for the sub-surface patch on which to look (may be omitted).

$\mathbf{x}_0$: a previous point on the CompositeSurface that is near to $\mathbf{x}$. This may be the previous location of $\mathbf{x}$ from a surface grid generator (may be omitted).

$\mathbf{n}_0$: normal to the CompositeSurface at the point $\mathbf{x}_0$ (may be omitted).

$\mathbf{x}_p$: projected point on the surface.

s_p : subsurface index where the point was projected.

$\mathbf{n}_p$: normal to the CompositeSurface at the point $\mathbf{x}_p$.

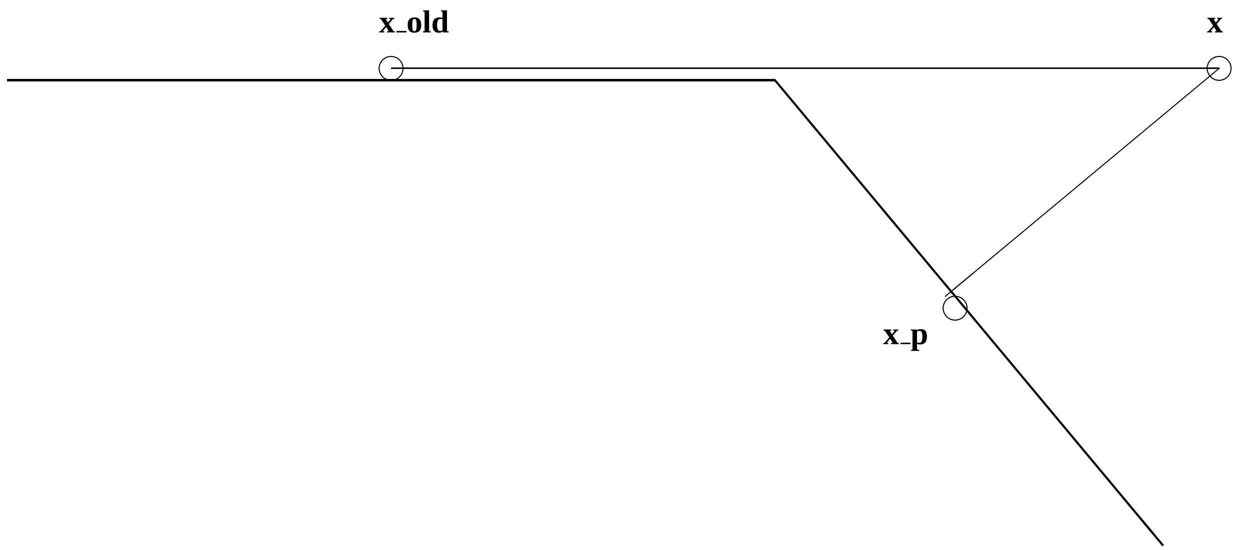
Here is the basic projection algorithm

1. Project $\mathbf{x}$ on the sub-surface patch s_0, giving the point $\mathbf{y} = P_{s_0}(\mathbf{x})$. If $\mathbf{y}$ is in the interior of the sub-surface then we are done.
2. If $\mathbf{y}$ is on the boundary of the sub-surface s_0 then compute the distance $\mathbf{d}_0 = \|\mathbf{x} - \mathbf{y}\|$. We will try to find a sub-surface that is closer than this distance.
3. Choose a new sub-surface to check, s_1, and project $\mathbf{x}$ onto this surface, $\mathbf{y}_1 = P_{s_0}(\mathbf{x})$.

9.1.1 Moving around sharp corners

If the point $\mathbf{x}$ to be projected is near a sharp corner in the surface then there is some ambiguity as to the desired projection point $\mathbf{x}_p$.

If we are marching over the surface then we usually want the projected point $\mathbf{x}_p$ to be some specified distance from the old point $\mathbf{x}_0$. In this case we may have to adjust the projected point and move it away from the corner.



9.2 Constructor

CompositeSurface()

Purpose: Default Constructor

9.3 operator =

CompositeSurface &
operator =(const CompositeSurface & X0)

Purpose: operator equal is a deep copy

9.4 add

int
add(Mapping & surface,
const int & surfaceID = -1)

Purpose: Add a surface to the composite surface

surface (input): add this mapping to the composite surface.

surfaceID (input): optional surface identification number. This could identify the surface in a CAD file, for example.

9.5 isVisible

int
isVisible(const int & surfaceNumber) const

Description: Query whether a sub-surface is visible.

surfaceNumber : sub-surface index from 0 to numberOfSubSurfaces()-1

9.6 setIsVisible

int
setIsVisible(const int & surfaceNumber,
const bool & trueOrFalse =TRUE)

Description: Set the visibity of a sub-surface. Invisible sub-surfaces are NOT considered by the project function.

surfaceNumber : sub-surface index from 0 to numberOfSubSurfaces()-1

trueOrFalse (input) : true if visible, else invisible.

9.7 findOutwardTangent

int
findOutwardTangent(Mapping & map, const realArray & r, const realArray & x, realArray & outwardTangent)

Access: This is a **protected** routine.

Purpose: Determine the outward tangent at point r on the edge of a (trimmed) sub-surface. If r is on the boundary of the unit square then it is easy to get the outward tangent. If r is near the the boundary of a trimmed surface then we find which trimming curve we on on and use the normal to the trimming curve (which is in r space) to get the outward tangent.

map (input): find the outward tangent of this Mapping.

r(0,0: 1) (input) : unit square coordinates on the surface.

x(0,0: 2) (input) : surface coordinates x=map(r)

outwardTangent(0,0: 2) : outward tangent (if return value==0)

Return values: 0 on success, 1 for failure.

9.8 findNearbySurfaces

void
findNearbySurfaces(const int & s,
realArray & r,
const bool & doubleCheck,
IntegerArray & consistent,
IntegerArray & inconsistent)

visibility: This is a private routine.

Description: Given a point r on the boundary of a surface, find any nearby surfaces to this point and set the signForNormal array

9.9 determineTopology

void
determineTopology()

Purpose: This is a private function. Determine some topology info about the composite surface: Determine the sign for each normal so that the normals of all surfaces are consistent.

Algorithm: We want to assign a value of +1 or -1 to each surface (signForNormal(s)) to indicate if we need to reverse the normal of the surface or not.

- surface zero is arbitrarily given a sign of +1. All other surfaces are given a unique positive number to identity the surface
- We now try to link surfaces together. If two surfaces are connected at a boundary then we assign the same number to them (actually plus or minus the same number depending on whether the normals need to be reversed). If the two surfaces are already connected to other surfaces then all connected surfaces get (+/-) the same value.

- If one of the surfaces is numbered +/- 1 (then it must be connected to surface zero) then all connected surfaces will get a value of +/- 1
- stop checking when all surfaces have a value of +/- 1

9.10 numberOfSubSurfaces

int
numberOfSubSurfaces() const

Purpose: return the total number of sub-surfaces that make up this composite surface

9.11 []

Mapping &
operator [](const int & subSurfaceIndex)

Purpose: return the Mapping that represents a subSurface

9.12 printStatistics

int
printStatistics(FILE *file =stdout)

Purpose: Print some statistics about the CompositeSurface. Currently only some timing statistics for the project function are presented.

9.13 remove

int
remove(const int & surfaceNumber)

Purpose: Remove a sub-surface from the composite surface

surfaceNumber (input): remove this surface.

9.14 recomputeBoundingBox

void
recomputeBoundingBox()

Purpose: Recompute the bounding box of the CompositeSUrface by querying all subsurfaces (visible and invisible) of their bounding boxes. Use this routine sparingly, since changing the bounding box will make the plot translate on the screen.

Author: AP

9.15 getColour

aString
getColour(const int & surfaceNumber) const

Purpose: Get the colour of a sub-surface.

surfaceNumber (input): sub-surface to set.

Return value : the name of the colour.

9.16 setColour

int
setColour(const int & surfaceNumber, const aString & colour)

Purpose: Set the colour for a sub-surface.

surfaceNumber (input): sub-surface to set.

colour (input) : the name of the colour such as ”red”, ”green”,...

9.17 project

int
project(realArray & x,
MappingProjectionParameters & mpParams)

Purpose: Project the points x(i,0:2) onto the surface. Also return the sub-surface index

subSurfaceIndex (input/output) : The index of the sub-surface that the point is closest to. On input this is the index of the previous point (if ¿= 0)

elementIndex (input/output) : if the CompositeSurface has an associated triangulation then this will be the closest element on the triangulation. On input this is a guess to the closest triangulation (if ¿=0).

x (input) : project these points onto the surface.

rProject (input/output) : sub-surface coordinates. On input these are an initial guess. On output they are the actual unit square coordinates.

xProject (input/output) : on input these are the projected points from the previous step (if subSurfaceIndex¿=0 on input). On output these are the projected points.

xrProject (output) : the derivative of the mapping at xProject

normal (input/output) : on input this is the normal to the surface at the old point. On output this array then it will hold the normal to the surface, normal(i,0:2). The normal vector will be chosen so that it is consistent across all sub-surfaces

ignoreThisSubSurface(i) (input) : Optional. Do not consider this sub-surface when projecting point x(i,0:2).

9.18 project

void
project(intArray & subSurfaceIndex,
realArray & x,
realArray & rProject,
realArray & xProject,
realArray & xrProject,
realArray & normal = Overture::nullRealDistributedArray(),
const intArray & ignoreThisSubSurface = Overture::nullIntArray(),
bool invertUntrimmedSurface = false)

Purpose: Project the points x(i,0:2) onto the surface. Also return the sub-surface index NOTE: invisible surfaces are ignored when projecting.

subSurfaceIndex (input/output) : The index of the sub-surface that the point is closest to. On input this is the index of the previous point (if ¿= 0)

x (input) : project these points onto the surface.

rProject (input/output) : sub-surface coordinates. On input these are an initial guess. On output they are the actual unit square coordinates.

xProject (input/output) : on input these are the projected points from the previous step (if subSurfaceIndex¿=0 on input). On output these are the projected points. These should always have some valid values on input to prevent purify UMR problems.

xrProject (output) : the derivative of the mapping at xProject

normal (input/output) : on input this is the normal to the surface at the old point. On output this array then it will hold the normal to the surface, normal(i,0:2). The normal vector will be chosen so that it is consistent across all sub-surfaces

ignoreThisSubSurface(i) (input) : Optional. Do not consider this sub-surface when projecting point x(i,0:2).

invertUntrimmedSurface: if tru only invert the untrimmed surface of a trimmed mapping. Use this option if the triangulation has already been used to find the closest sub-surface.

9.19 map

void
map(const realArray & r, realArray & x, realArray & xr, MappingParameters & params)

Purpose: This routine should not normally be called

9.20 getSignForNormal

int
getSignForNormal(int s) const

Description: Return the sign of the normal for sub-surface s, either +1 or -1; In order to orient the normals to the sub-surfaces in the same direction it may be necessary to reverse the normals of some sub-surfaces.

9.21 setTolerance

int
setTolerance(real tol)

Description: Set the tolerance for how well the surfaces match (may come from the CAD file)

9.22 getTolerance

real
getTolerance() const

Description: Get the tolerance for how well the surfaces match (may come from the CAD file)

9.23 eraseCompositeSurface

void
eraseCompositeSurface(GenericGraphicsInterface &gi, int surface = -1)

Description: purge all display lists if surface = -1, otherwise, just purge one list

surface (input): purge the display lists for this surface. By default purge all lists.

9.24 findBoundaryCurves

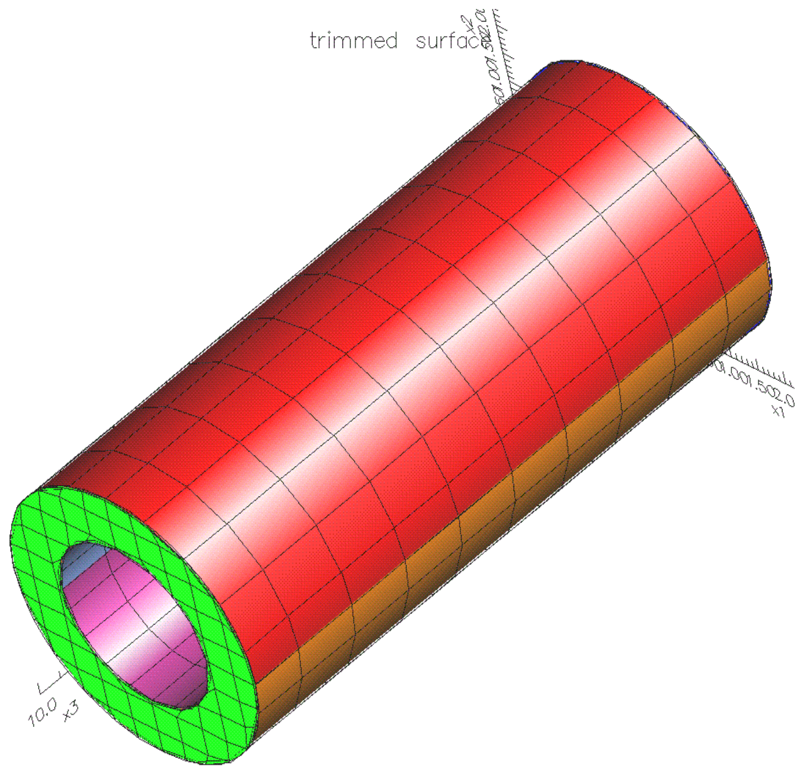
int
findBoundaryCurves(int & numberOfBoundaryCurves, Mapping **& boundaryCurves)

Description: Locate boundary curves on a CompositeSurface. Merge boundary edge curves that form a smooth portion of the boundary.

numberOfBoundaryCurves (output) : number of boundary curves found.

boundaryCurves (output) : Boundary curves.

9.25 Examples



A CompositeSurface for a cylindrical surface read from an IGES file created by pro/ENGINEER

10 CrossSectionMapping: define various surfaces by cross-sections

10.1 Description

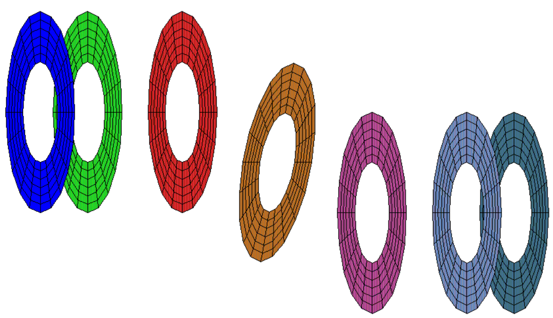
The CrossSectionMapping can be used to define a Mapping from a collection of cross-sectional curves or surfaces. The available options for the cross-section type are

general: Build a mapping from a sequence of cross sections. The cross sections may be curves (such as circles or splines etc.) or they may be surfaces such as an Annulus or SmoothPolygonMapping.

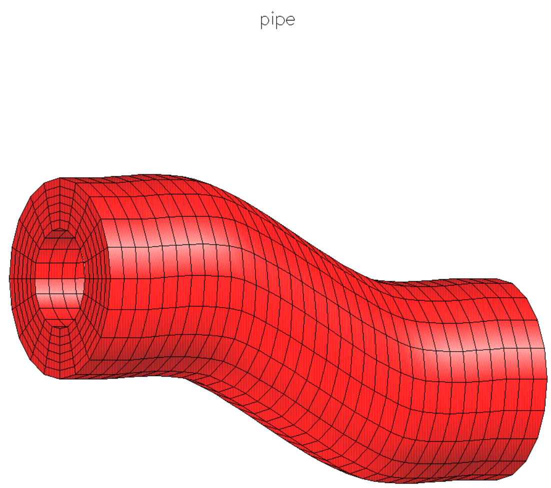
ellipse: Define an ellipsoid, either a surface or a shell.

joukowsky: Define a "wing" surface with cross sections defined as Joukowsky airfoils.

Thanks go to Thomas Rutaganira for help with this Mapping.



Six AnnulusMapping's to be used as cross-sections.



A volume grid created from the six AnnulusMapping's (cubic interpolation).

10.2 General cross-section type

When the cross-section type is `general` the user specifies a sequence of curves (or surfaces) that will be used as the cross-sections.

Given a sequence of n cross-sectional curves

$$\mathbf{c}_i(r_0), \quad i = 0, 1, ..., n-1 \qquad \text{(cross-section curves)}$$

the `CrossSectionMapping` defines a surface by blending the curves in the regions between them.

$$\mathbf{x}(\mathbf{t}, r_a) = \mathbf{C}(\mathbf{c}_i(\mathbf{t}), r_a) \; .$$

The parameter direction(s) $\mathbf{t}$ will be called the *tangential* direction(s). If the cross-sections are curves then $\mathbf{t} = r_0$; if they are surfaces $\mathbf{t} = (r_0, r_1)$. The direction r_a will be called the *axial* direction. As r_a varies for fixed $\mathbf{t}$ we trace a curve that follows the axis of the surface.

With linear interpolation (default) the curve is a linearly interpolated between succussive cross-sections:

$$\mathbf{x}(\mathbf{t}, r_a) = (1 - s_a)\mathbf{c}_i(\mathbf{t}) + s_a \mathbf{x}_{i+1}(\mathbf{t}) \qquad \text{for } \frac{i}{n-1} \le S(r_a) \le \frac{i+1}{n-1}$$
$$s_a = S(r_a)(n-1) - \lfloor S(r_a)(n-1) \rfloor$$

where the axial parameterization function $S(r_a)$ is defined below. The variable s_a varies between 0 and 1 as we move from cross-section i to cross-section $i+1$. Here $\lfloor x \rfloor$ is the biggest integer less or equal to x.

With `index` parameterization $S(r_a) = r_a$ in which case the cross-sections are parameterized as if they were equally spaced. Thus there will be approximately an equal number of axial grid lines between any two cross-sections. Normally this is not a good parameterization unless the cross-sections are nearly equally spaced.

With `arcLength` parameterization (the default) the axial direction is parameterized using the average distance between the cross-sectional curves. The average distance between the curves is computed by evaluating each curve at m equally spaced points $\{\mathbf{c}_i(t_j)\}_{j=0}^{m-1}$, $t_j = j/(m-1)$, and then taking the average of the distances between these points.

$$
\begin{aligned}
s_{i+1} &= s_i + \|\mathbf{c}_{i+1} - \mathbf{c}_i\|/L \qquad s_0 = 0, \quad \text{L chosen so } s_{n-1} = 1. \\
\|\mathbf{x}_{i+1} - \mathbf{c}_i\| &= \frac{1}{m} \sum_{j=0}^{m-1} \|\mathbf{x}_{i+1}(t_j) - \mathbf{c}_i(t_j)\|
\end{aligned}
$$

The *inverse* of the function $S(r_a)$ is defined by fitting a spline to the data points $\{s_i\}_{i=0}^{n-1}$. That is a spline fitted to the points $\{s_i\}_{i=0}^{n-1}$ will define the function S^{-1}. The exact properties of the spline can be adjusted by choosing the "`change arclength spline parameters`" option. For example, one may want to use a spline with tension or a spline that is shape preserving. See the SplineMapping documentation, section (33), for further details.

With a `userDefined` parameterization the user defines the parameter values s_i for each of the cross-sections. The values s_i should satisfy $s_0 = 0$, $s_i < s_{i+1}$ and $s_{n-1} = 1$. Normally the value of $s_i - s_{i-1}$ would be based on the distance between the cross-section curves $i-1$ and i. The inverse of the function $S(r_a)$ is defined by fitting a spline to the data points $\{s_i\}_{i=0}^{n-1}$.

With piecewise cubic interpolation the mapping is defined as a cubic polynomial on each interval (except the first and last where quadratic polynomials are used)

$$
\begin{aligned}
\mathbf{x}(\mathbf{t}, r_a) &= q_{03}(s_a)\mathbf{c}_{i-1}(\mathbf{t}) + q_{13}(s_a)\mathbf{x}_i(\mathbf{t}) + q_{23}(s_a)\mathbf{x}_{i+1}(\mathbf{t}) + q_{33}(s_a)\mathbf{x}_{i=2}(\mathbf{t}) \qquad \text{for } \frac{i}{n-1} \le S(r_a) \le \frac{i+1}{n-1} \\
s_a &= S(r_a)(n-1) - \lfloor S(r_a)(n-1) \rfloor
\end{aligned}
$$

where q_{i3} are cubic Lagrange polynomials. On the left edge a quadratic polynomial is used which passes through the cross sections $0, 1, 2$. Similarly for the right edge.

10.2.1 Notes for generating general cross section mappings

1. For best results the cross sections should be **nearly equally spaced**.
2. With the cubic interpolation option: quadratic polynomials are used on the first and last segments. If you wish an end segment to be "straight" then you should place three cross sections in a straight line at the end.
3. It is up to you to make sure that the cross sections are all parameterized in a compatible fashion; if they are not then the axial grid lines may twist and the grid may not be invertible.
4. With the cubic interpolation option: if the cross sections vary rapidly from one to the next or the cross sections are very unevenly spaced then the cubic interpolant (or quadratic interpolants on the ends) may wiggle a lot. Adding more cross-sections should fix this problem.

10.3 Ellipse cross-section type

When the cross-section type is `ellipse` the Mapping defines an ellipsoid in cylindrical coordinates with semi-axes `a,b,c`:

$$
\begin{aligned}
\zeta &= (\mathrm{endS} - \mathrm{startS})\mathrm{r}_0 - (1. - 2. * \mathrm{startS}) \\
\rho &= \sqrt{1 - \zeta^2} \\
R &= \mathrm{innerRadius} + r_2(\mathrm{outerRadius} - \mathrm{innerRadius}) \\
x_0 &= aR\rho\cos(2\pi r_1) + \mathrm{x0} \\
x_1 &= bR\rho\sin(2\pi r_1) + \mathrm{y0} \\
x_2 &= cR\zeta + \mathrm{z0}
\end{aligned}
$$

The default values for the parameters are startS=0, endS=1, innerRadius=1, outerRadius=1.5, x0=0, y0=0, z0=0.

After building an ellipsoid one would normally remove the singularities at the poles by building patches to cover the ends using the `reparameterize` option. See the example in the overlapping grid documentation.

10.4 Joukowsky cross-section type

This section needs to be written.

10.5 Cross section Mappings with polar singularities

It is often the case that one desires the cross-sections to converge to a point at one or both ends. In this case one should indicate that the Mapping has a polar singularity at one or both ends. One should also choose the last cross section to be a small ellipse. The CrossSectionMapping will then slightly deform the Mapping to cause the last cross-section to converge to a point. The resulting deformed Mapping can then have an orthographic patch built to cover the singularity using the `ReparameterizationTransform`.

In order for the `OrthographicTransform` to nicely remove a polar singularity, the Mapping with the singularity must locally near the pole be parameterized like

$$\begin{aligned}
&\mathbf{x} \sim A\rho(r_1)(a\cos(\theta(r_2), b\sin(\theta(r_2))) \\
&\rho = \sqrt{1-\zeta^2} \\
&\zeta = 2r_1 - 1
\end{aligned}$$

Thus locally the surface must look like an ellipsoid (it can be oriented in any direction, the above equation assumes a particular orientation). The "radius" of the cross section, defined, say, by the average distance of the cross-section from its centroid, should be decaying like $\rho \sim \sqrt{r_1}$ as $r_1 \rightarrow 0$. If the radius decays at a different rate then the coordinates lines on the orthographic patch will not be rectangular near the pole.

10.6 Constructor

CrossSectionMapping(
const real startS_,
const real endS_,
const real startAngle_,
const real endAngle_,
const real innerRadius_ ,
const real outerRadius_,
const real x0_ ,
const real y0_ ,
const real z0_ ,
const real length_,
const int domainDimension_)

Description: Default Constructor, define a mapping from cross-sections.

Build a mapping defined by cross sections. In the `general` case the cross-sections are defined by other Mapping's. One can also build an ellipsoid when the cross-section type is `ellipse` or a Joukowsky wing. enum CrossSectionType:

- general
- ellipse
- joukowsky

enum Parameterization:

arcLength : parameterize by the arc length distance between the centroids of the cross sectional curves.

index : parameterize by the index of the cross section.

userDefined : supply a parameterization.

10.7 setCrossSectionType

int
setCrossSectionType(CrossSectionTypes type)

Description: Define the cross-section type. *this is not finished yet*

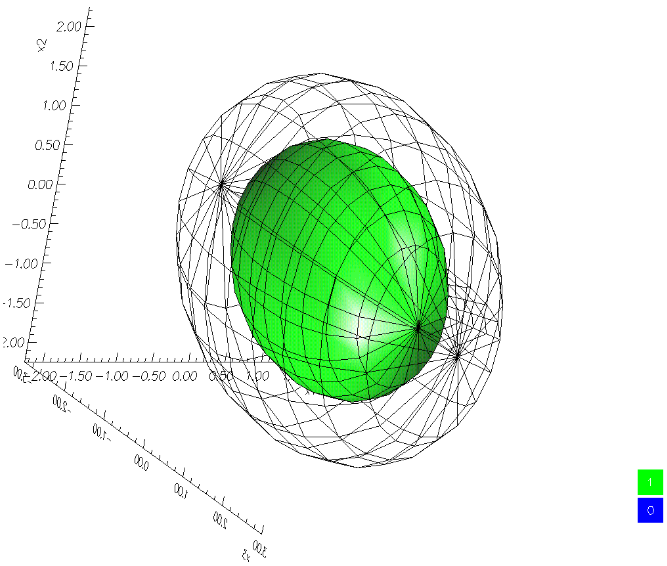
type (input):

10.8 Constructor

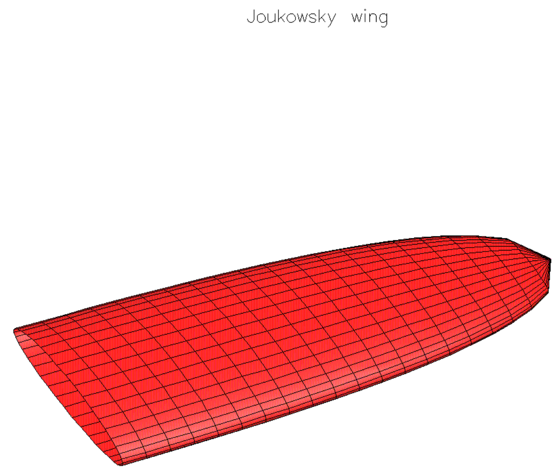
int
initialize()

Description: private routine. Initialize the parameterization for the cross sections.

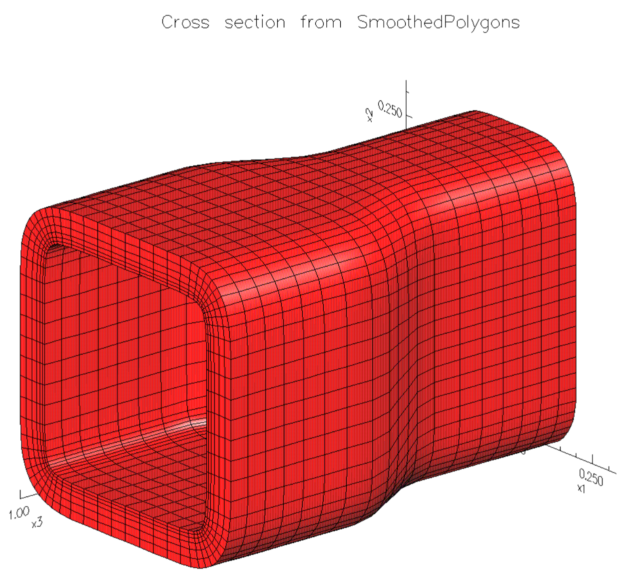
10.9 Examples



An ellipsoid created with the `ellipse` cross-section type.

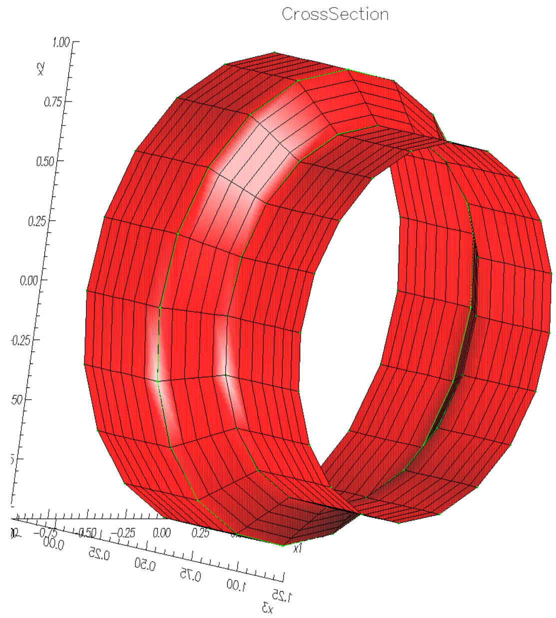


A Joukoswky airfoil created with the `joukowsky` cross-section type.

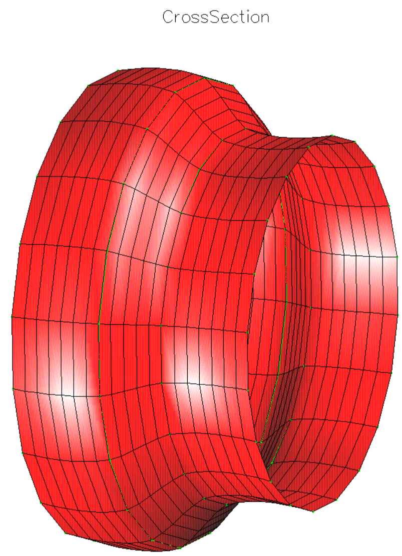


A volume grid created from 4 smoothed polygon cross-section surfaces (linear interpolation).

```
*
* Define a cross section mapping
*
Circle or ellipse (3D)
  specify radius of the circle
    1.
  specify centre
    0. 0. 0.
  lines
    21  25
  mappingName
   circle0
exit
Circle or ellipse (3D)
  specify radius of the circle
    1.
  specify centre
    0. 0. .4
  lines
    21  31
  mappingName
   circle1
exit
Circle or ellipse (3D)
  specify radius of the circle
    .8
  specify centre
    0. 0. .6
  mappingName
   circle2
exit
Circle or ellipse (3D)
  specify radius of the circle
    .8
  specify centre
    0. 0. 1.
  mappingName
   circle3
exit
*
* make a cross section mapping
CrossSection
  general
  4
  circle0
  circle1
  circle2
  circle3
  x+r 30
  y+r
```



A surface grid created from 4 circular cross-sections (linear interpolation). The cross-sections are shown in green.



As above with cubic interpolation.

11 CylinderMapping

This mapping defines a cylindrical volume or surface in three-dimensions.

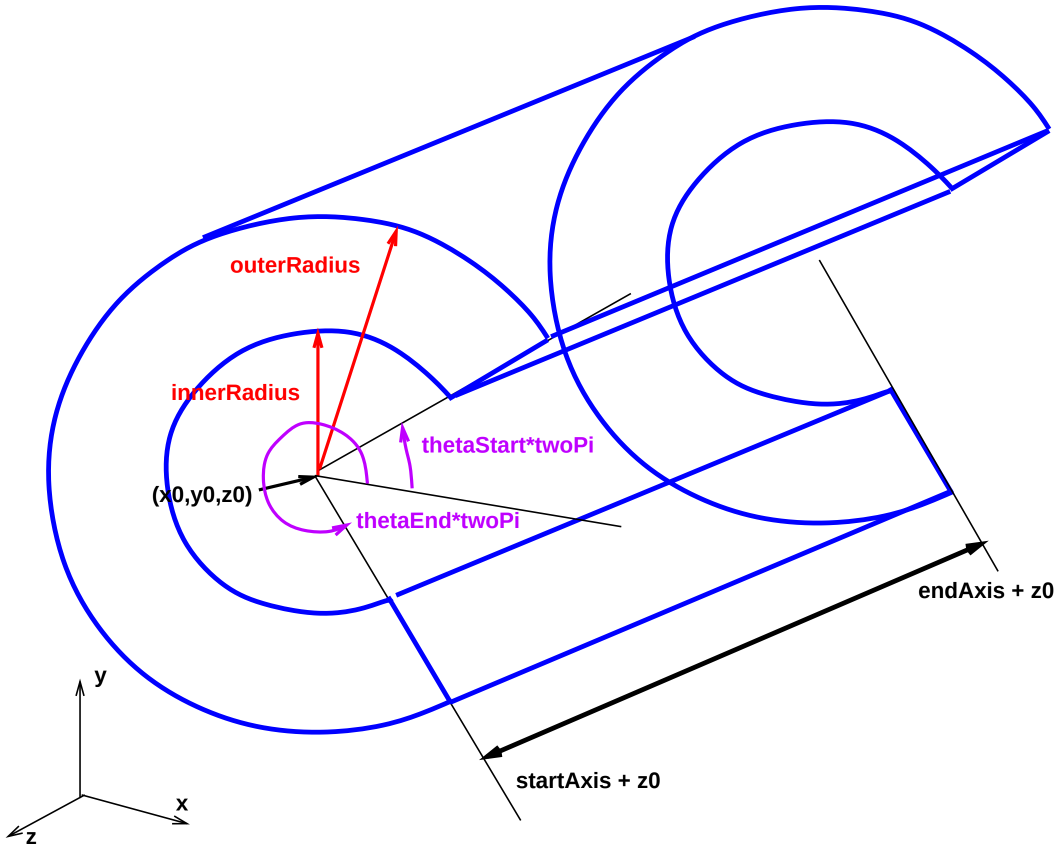


Figure 4: The CylinderMapping defines a cylinder in three-dimensions.

11.1 Constructor

CylinderMapping(
const real & startAngle_ = 0.,
const real & endAngle_ = 1.,
const real & startAxis_ = -1.,
const real & endAxis_ = +1.,
const real & innerRadius_ = 1.,
const real & outerRadius_ = 1.5,
const real & x0_ = 0.,
const real & y0_ = 0.,
const real & z0_ = 0.,
const int & domainDimension_ = 3,
const int & cylAxis1_ = axis1,
const int & cylAxis2_ = axis2,
const int & cylAxis3_ = axis3
)

Purpose: Create a 3D cylindrical volume or surface.

Notes: This mapping defines a cylinder in three-dimensions:

$$\theta = 2\pi(\theta_0 + r_0(\theta_1 - \theta_0))$$

$$R(r_1) = (R_0 + r_1(R_1 - R_0))$$
$$\mathbf{x}(r_0, r_1, r_2) = (R\cos(\theta) + x_0, R\sin(\theta) + y_0, s_0 + r_2(s_1 - s_0) + z_0)$$

The above cylinder has the z-axis as the axial direction. It is also possible to to have the axial direction to point in any of the coordinate direction using the (`cylAxis1`, `cylAxis2`, `cylAxis3`) variables (which should be a permutation of (0,1,2)):

$$\mathbf{x}(r_{cylAxis1}, r_{cylAxis2}, r_{cylAxis3}) = (R\cos(\theta) + x_0, R\sin(\theta) + y_0, s_0 + r_2(s_1 - s_0) + z_0)$$

startAngle (input) : starting angle (θ_0) NOTE: angles are 1-periodic!

endAngle (input) : ending angle (θ_1) NOTE: angles are 1-periodic!.

startAxis (input) : axial coordinate of the start of the cylinder (s_0).

endAxis (input) : axial coordinate of the end of the cylinder (s_1).

innerRadius (input) : inner radius (R_0).

outerRadius (input) : outer radius (R_0).

x0,y0,z0 (input) : center of the cylinder (x_0,y_0,z_0).

domainDimension (input) : 3 means the cylinder is a volume, 2 means the cylinder is a surface.

cylAxis1,cylAxis2,cylAxis3 (input) : change these to be a permutation of (axis1,axis2,axis3) to change the orientation of the cylinder. NOTE: axis1==0, axis2==1, axis3==2.

11.2 setAngle

int
setAngle(const real & startAngle_ =0.,
const real & endAngle_ =1.)

Description: Set the initial and final angles.

startAngle (input) :

endAngle (input) :

11.3 setAxis

int
setAxis(const real & startAxis_ =-1.,
const real & endAxis_ =+1.)

Description: Set the starting and ending axial positions.

startAxis (input) : axial coordinate of the start of the cylinder (s_0).

endAxis (input) : axial coordinate of the end of the cylinder (s_1).

11.4 setOrientation

int
setOrientation(const int & cylAxis1_ =0,
const int & cylAxis2_ =1,
const int & cylAxis3_ =2)

Description: Set the orientation of the cylinder.

cylAxis1,cylAxis2,cylAxis3 (input) : change these to be a permutation of (axis1,axis2,axis3) to change the orientation of the cylinder. NOTE: axis1==0, axis2==1, axis3==2.

11.5 setOrigin

int
setOrigin(const real & x0_ =0.,
const real & y0_ =0.,
const real & z0_ =0.)

Description: Set the centre of the cylinder.

x0,y0,z0 (input) : center of the cylinder (x_0,y_0,z_0).

11.6 setRadius

int
setRadius(const real & innerRadius_ =1.,
const real & outerRadius_ =1.5)

Description: Set the inner and outer radii.

innerRadius (input) : inner radius (R_0).

outerRadius (input) : outer radius (R_0).

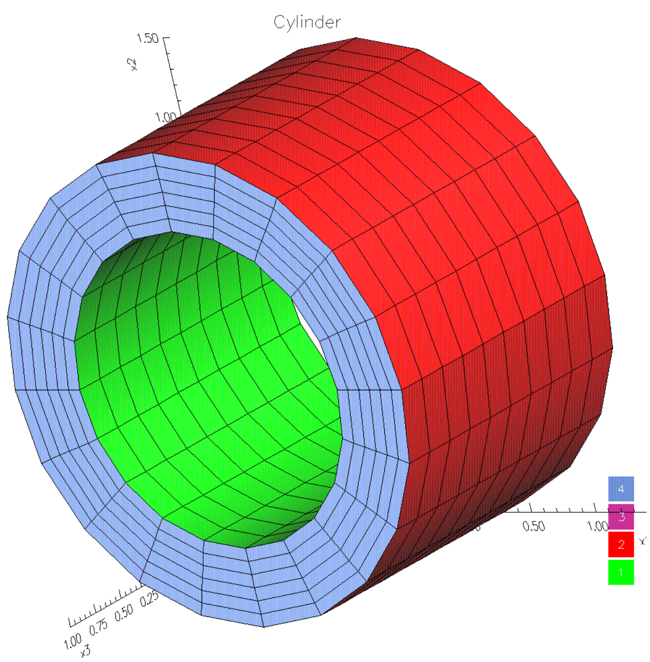


Figure 5: CylinderMapping. This is the volume representation. The cylinder may also represent a surface.

12 DataPointMapping: create a mapping from an array of grid points

12.1 Description

The `DataPointMapping` can be used to create a mapping from a set of grid points. The grid points may be in a file (such as Plot3D format) or can be in an A++ array.

The `DataPointMapping` defines a Mapping transformation by interpolating the grid points.

For `orderOfInterpolation=2` the transformation is defined as a linear interpolation (i.e. 2 points in each direction). In 2D this would be

$$\mathbf{x}(\mathbf{r}) = (1-\hat{r}_2)[(1-\hat{r}_1)\mathbf{x}_{00} + \hat{r}_1\mathbf{x}_{10}] + \hat{r}_2[(1-\hat{r}_1)\mathbf{x}_{01} + \hat{r}_1\mathbf{x}_{11}]$$

where $\mathbf{x}_{mn}$ are the grid points that define the cell and $\hat{r}_m$ are the relative distances of the point $\mathbf{r}$ from the r-coordinates of the corner point $\mathbf{x}_{00}$

$$\hat{r}_1 = \frac{(r_1 - r_{00})}{\Delta r_1} \quad , \quad \hat{r}_2 = \frac{(r_2 - r_{00})}{\Delta r_2}$$

The derivatives returned by the Mapping are just the derivatives of the above expression.

For `orderOfInterpolation=4` the transformation is defined as a cubic interpolation (i.e. 4 points in each direction). In 2D this would be defined as

$$\mathbf{x}(\mathbf{r}) = \sum_{m=0}^{3} q_m(\hat{r}_2) \sum_{n=0}^{3} q_n(\hat{r}_1)\mathbf{x}_{nm} \tag{1}$$

$$q_m(s) = \prod_{n\neq m} (s-n)/(m-n) \qquad \text{Lagrange polynomials,} \quad q_m(n) = \delta_{mn} \tag{2}$$

The derivatives returned by the Mapping are just the derivatives of the above expression.

Figure 12.1 shows a grid for part of the coast of the USA, created by Lotta Olsson. The grid was created with the help of the HYPGEN hyperbolic grid generator and saved in Plot3D format.

12.2 Fast Approximate Inverse

Since the DataPointMapping is extensively used, a specialized fast approximate inverse has been defined (not yet!) for linear interpolation.

The inverse consists of the steps:

1. Find the closest vertex on the grid to the point, $\mathbf{x}$, to be inverted.
2. Find the hexahedral that $\mathbf{x}$ is in.
3. Invert the bilinear (tri-linear) mapping (approximately).

The linear interpolant within a given cell is

$$\mathbf{x}(\mathbf{r}) = (1-\hat{r}_0)[(1-\hat{r}_1)\mathbf{x}_{00} + \hat{r}_1\mathbf{x}_{01}] + \hat{r}_0[(1-\hat{r}_1)\mathbf{x}_{10} + \hat{r}_1\mathbf{x}_{11}]$$

in 2D or

$$\begin{aligned}\mathbf{x}(\mathbf{r}) = (1-\hat{r}_0)&[(1-\hat{r}_1)((1-\hat{r}_2)\mathbf{x}_{000} + \hat{r}_2\mathbf{x}_{001}) + \hat{r}_1((1-\hat{r}_2)\mathbf{x}_{010} + \hat{r}_2\mathbf{x}_{011})] \\ &+\hat{r}_0[(1-\hat{r}_1)((1-\hat{r}_2)\mathbf{x}_{100} + \hat{r}_2\mathbf{x}_{101}) + \hat{r}_1((1-\hat{r}_2)\mathbf{x}_{110} + \hat{r}_2\mathbf{x}_{111})]\end{aligned}$$

in 3D where $\mathbf{x}_{lmn}$ are the grid points that define the cell and $\hat{r}_m$ are the scaled unit square coordinates, $\hat{r}_m \in [0,1]$ for points within the cell.

A Newton iteration to invert the mapping would look like

$$\mathbf{x}(\mathbf{r}_0 + \delta\mathbf{r}) = \mathbf{x}(\mathbf{r}_0) + \frac{\partial\mathbf{x}}{\partial\mathbf{r}}(\mathbf{r}_0)\delta\mathbf{r}.$$

where

$$\left[\frac{\partial\mathbf{x}}{\partial\mathbf{r}}\right] = \begin{bmatrix}\mathbf{a}_0 & \mathbf{a}_1 & \mathbf{a}_2\end{bmatrix}$$

where

$$
\begin{aligned}
\mathbf{a}_0 &= (1-\hat{r}_1)((1-\hat{r}_2)(\mathbf{x}_{100}-\mathbf{x}_{000}) + \hat{r}_2(\mathbf{x}_{101}-\mathbf{x}_{001})) + \hat{r}_1((1-\hat{r}_2)(\mathbf{x}_{110}-\mathbf{x}_{010}) + \hat{r}_2(\mathbf{x}_{111}-\mathbf{x}_{011})) \\
\mathbf{a}_1 &= (1-\hat{r}_0)((1-\hat{r}_2)(\mathbf{x}_{010}-\mathbf{x}_{000}) + \hat{r}_2(\mathbf{x}_{011}-\mathbf{x}_{001})) + \hat{r}_1((1-\hat{r}_2)(\mathbf{x}_{110}-\mathbf{x}_{100}) + \hat{r}_2(\mathbf{x}_{111}-\mathbf{x}_{101})) \\
\mathbf{a}_2 &= (1-\hat{r}_0)((1-\hat{r}_1)(\mathbf{x}_{001}-\mathbf{x}_{000}) + \hat{r}_2(\mathbf{x}_{101}-\mathbf{x}_{100})) + \hat{r}_1((1-\hat{r}_2)(\mathbf{x}_{011}-\mathbf{x}_{010}) + \hat{r}_2(\mathbf{x}_{111}-\mathbf{x}_{110}))
\end{aligned}
$$

In the special case when the cell is a regular 'diamond' shape the linear interpolant simplifies to

$$
\mathbf{x}(\mathbf{r}) - \mathbf{x}_{000} = \hat{r}_0(\mathbf{x}_{100}-\mathbf{x}_{000}) + \hat{r}_1(\mathbf{x}_{010}-\mathbf{x}_{000}) + \hat{r}_2(\mathbf{x}_{001}-\mathbf{x}_{000})
$$

or

$$
\mathbf{x}(\mathbf{r}) = \hat{r}_0\mathbf{x}_{100} + \hat{r}_1\mathbf{x}_{010} + \hat{r}_2\mathbf{x}_{001}
$$

where the 'inverse' is computed by solving a $d \times d$ matrix

$$
\begin{bmatrix} \mathbf{x}_{100} & \mathbf{x}_{010} & \mathbf{x}_{001} \end{bmatrix} \begin{bmatrix} \hat{r}_0 \\ \hat{r}_1 \\ \hat{r}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{x} \end{bmatrix}
$$

** The newton step will do this case exactly**

If the cell is not regular we can compute an approximate inverse by first computing coordinates $\hat{\mathbf{r}}^{lmn}$ for each vertex, assuming a regular diamond shape at the vertex $\mathbf{x}_{lmn}$ formed from the points that connect to the vertex.

12.3 Constructor

DataPointMapping()

Purpose: Default Constructor.

12.4 getDataPoints

getDataPoints()

Description: Return the array of data points. It will not be the same array as was given to setDataPoints since ghostlines will have been added. Use getGridIndexRange to determine the index positions for the grid boundaries.

Return value: array of data points, xy(I1,I2,I3,0:r-1), r=rangeDimension

12.5 getGridIndexRange

const IntegerArray &
getGridIndexRange()

Description: Return the gridIndexRange array for the data points. These values indicate the index positions for the grid boundaries.

Return value: The gridIndexRange(0:1,0:2).

12.6 getDimension

const IntegerArray &
getDimension()

Description: Return the dimension array for the data points. These values indicate the index positions for the array dimensions.

Return value: The dimension(0:1,0:2).

12.7 setDataPoints

int
setDataPoints(const realArray & xd,
const int positionOfCoordinates =3,
const int domainDimension_ =-1,
const int numberOfGhostLinesInData = 0,
const IntegerArray & xGridIndexRange = Overture::nullIntArray())

Purpose: Supply data points as

1. xd(0:r-1,I,J,K) if positionOfCoordinates==0 → domainDimension=domainDimension_
2. xd(I,0:r-1) if positionOfCoordinates==1 → domainDimension=1
3. xd(I,J,0:r-1) if positionOfCoordinates==2 → domainDimension=2
4. xd(I,J,K,0:r-1) if positionOfCoordinates==3 → domainDimension=domainDimension_

where r=number of dimensions (range dimension)

xd (input): An array of values defining the coordinates of a grid of points. This routine make a COPY of this array.

positionOfCoordinates (input): indicates the "shape" of the input array xd.

domainDimension_ (input): As indicated above this parameter defines the domainDimension when positionOfCoordinates is 0 or 3.

numberOfGhostLinesInData (input) : The data includes the coordinates of this many ghost lines (for all sides). These values are over-ridden by the index array argument.

xGridIndexRange (input): If this array is not null and size (2,0:r-1) then these values indicate the points in the array xd that represent the boundary points on the grid. Use this option to specify arbitrary number of ghost points on any side.

Remarks: Note that by default the DataPointMapping will have the properties

- domainSpace = parameterSpace
- rangeSpace = cartesianSpace
- not periodic
- boundary conditions all 1

You will have to change the above properties as appropriate. NOTE: you should set the periodicity of this mapping before supplying data points.

12.8 setDataPoints

int
setDataPoints(const realArray & xd,
const int positionOfCoordinates,
const int domainDimension_,
const int numberOfGhostLinesInData[2][3],
const IntegerArray & xGridIndexRange = Overture::nullIntArray())

Description: Supply data points: Same as above routine except that the numberOfGhostLinesInData can be defined as separate values for each face.

numberOfGhostLinesInData[side][axis] : specify the number of ghostlines in the input data for each face.

12.9 computeGhostPoints

int
computeGhostPoints(int numberOfGhostLinesOld[2][3],
int numberOfGhostLinesNew[2][3])

Access Level: protected

Description: Determine values at ghost points that have not been user set: extrapolate or use periodicity Ghost lines on sides with boundaryCondition¿0 are extrapolated with a stretchingFactor (see below) so that the grid lines get further apart. This is useful for highly stretched grids so that the ghost points move away from the boundary.

12.10 setNumberOfGhostLines

int
setNumberOfGhostLines(int numberOfGhostLinesNew[2][3])

Description: Specify the number of ghost lines.

numberOfGhostLinesNew[side][axis] : specify the number of ghostlines.

12.11 projectGhostPoints

int
projectGhostPoints(MappingInformation & mapInfo)

Description: Project the ghost points on physical boundaries onto the closest mapping found in a list of Mapping's

mapInfo (input): Project onto the closest mapping found in mapInfo.mappingList.

12.12 setDataPoints(fileName)

int
setDataPoints(const aString & fileName)

Description: Assign the data points from a file of data. By default this routine will attempt to automaticall determine the format of the file.

fileName (input) : name of an existing file of data (such as a plot3d file)

12.13 setMapping

int
setMapping(Mapping & map)

Description: Build a data point mapping from grids points obtained by evaluating a mapping.

map (input) : Mapping to get data points from.

12.14 setOrderOfInterpolation

void
setOrderOfInterpolation(const int order)

Purpose: Set the order of interpolation, 2 or 4.

order (input) : A value of 2 or 4.

12.15 setOrderOfInterpolation

int
getOrderOfInterpolation()

Purpose: Get the order of interpolation.

Return value: The order of interpolation.

12.16 useScalarArrayIndexing

void
useScalarArrayIndexing(const bool & trueOrFalse =FALSE)

Purpose: Turn on or off the use of scalar indexing. Scalar indexing for array operations can be faster when the length of arrays are smaller.

trueOrFalse (input) : TRUE means turn on scalra indexing.

12.16.1 sizeOf

real
sizeOf(FILE *file = NULL) const

Description: Return size of this object

12.17 update

int
update(MappingInformation & mapInfo)

Purpose: Interactively change parameters describing the Mapping. The user may choose to read in data points from a file. The current supported file formats are

- plot3d

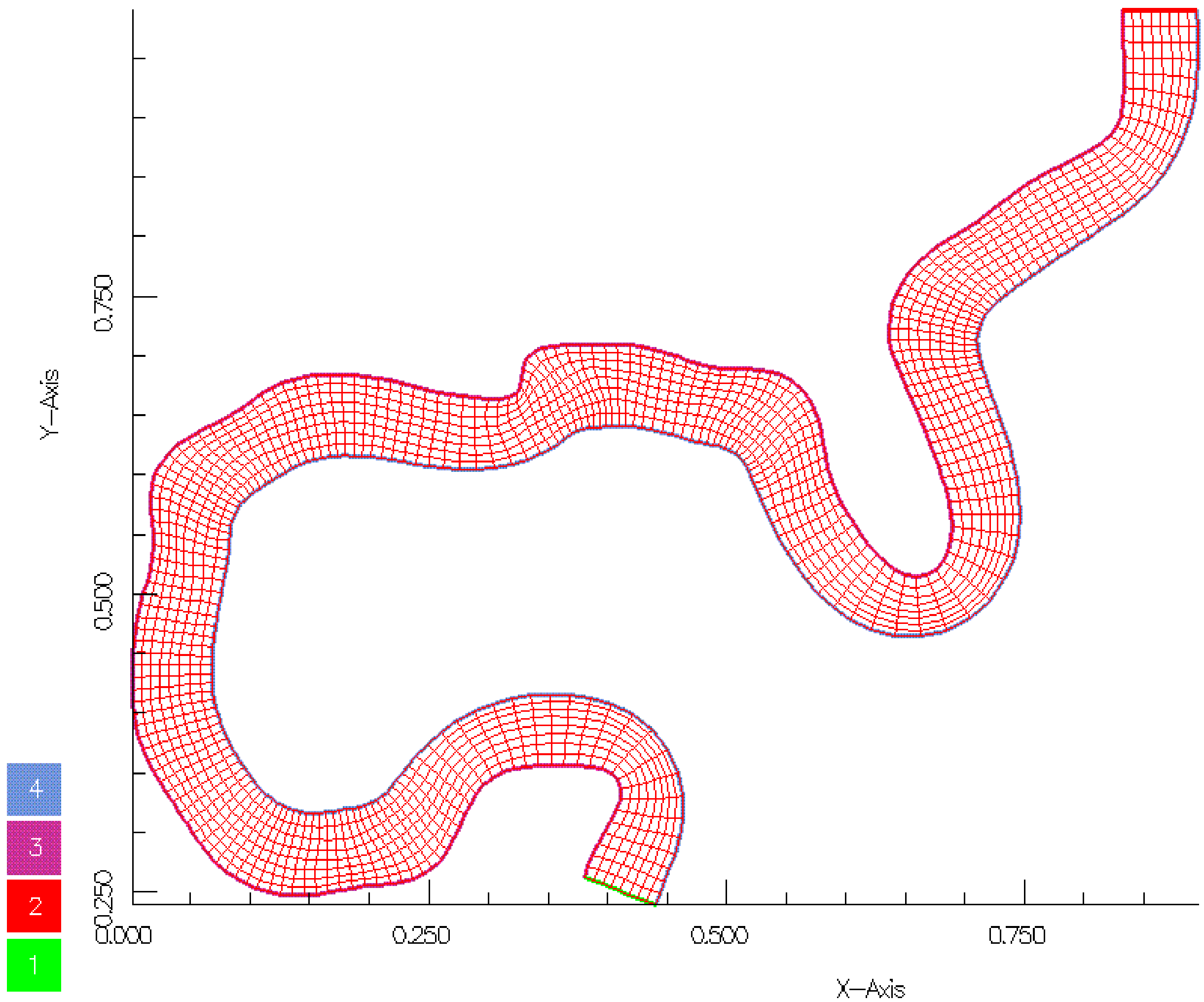


Figure 6: A DataPointMapping created from a Plot3D file.

13 DepthMapping: Add a depth to a 2D Mapping

13.1 Description

Define a 3D Mapping from 2D Mapping by extending in the z-direction by a variable amount

The depth mapping starts with some region defined in the $(x, y) = (x_0, x_1)$ plane, $(x_0, x_1) = \mathbf{x}_s(r_0, r_1)$, such as an annulus or square etc. It then defines a 3D volume (or 3D surface) of the form

$$\mathbf{x}(r_0, r_1, r_2) = (x_0(r_0, r_1), x_1(r_0, r_1), x_2(r_0, r_1, r_2))$$

As the variable r_2 varies, the initial 2D surface is deformed to define a generalized cylinder.

The depth coordinate, x_2 is defined in a number of ways:

Representation I: In this case the depth is a function of (x_0, x_1) and uses some predefined functions available in the `DepthMapping`,

$$\mathbf{x}(\mathbf{r}) = (\mathbf{x}_s(r_0, r_1), z_0 + r_2 z(\mathbf{x}_s(r_0, r_1)))$$

Representation II: depthFunction

One would like to be able to provide a depth function $z = d(x, y)$ which gives a depth as a function of (x, y). This is not easily done with the existing Mapping's in Overture, since most Mapping's are parameterized as transformations from the unit square, $\mathbf{x} = \mathbf{x}(\mathbf{r})$. Thus instead of a function $z = d(x, y)$, we use a parameterized representation,

$$\mathbf{d}(r_0, r_1, r_2) = (d_0(r_0, r_1), d_1(r_0, r_1), d_2(r_0, r_1, r_2)).$$

where only $d_2(r_0, r_1, r_2)$ is of interest. Now given a point (x, y) on the original 2D Mapping we want to determine the corresponding value for d_2. To do this we need a transformation from (x, y) to the arguments (r_0, r_1) of g_2, which we take to be the simple linear transformation $(x, y) = \mathbf{g}(x, y) = (a_0 + b_0 r_0, a_1 + b_1 r_1)$.

The volume depth mapping is then defined as

$$\mathbf{x}(r_0, r_1, r_2) = (x_0(r_0, r_1), x_1(r_0, r_1), d_2((x_0(r_0, r_1) - a_0)/b_0, (x_1(r_0, r_1) - a_1)/b_1, r_2))$$

A surface depth Mapping would simply omit the argument r_2. The scale parameters (a_0, b_0, a_1, b_1) must be supplied by the user to ensure that the scaled coordinates

$$\tilde{r}_i = (x_i - a_i)/b_i \qquad i = 0, 1$$

satisfy $0 \leq \tilde{r}_i \leq 1$ for all points $\mathbf{x}_s(r_0, r_1)$.

Normally the scale parameters just indicate how to scale fron the unit square into (x, y) corrdinates,

$$x_i = a_i + b_i r_i$$

So that if the physical domain covers the rectangle $[-1, 1] \times [-2, 2]$ one should take $a_0 = -1, b_0 = 2, a_1 = -2, b_1 = 4$ since $x = -1 + 2 * r_0$, $y = -2 + 4 * r_1$.

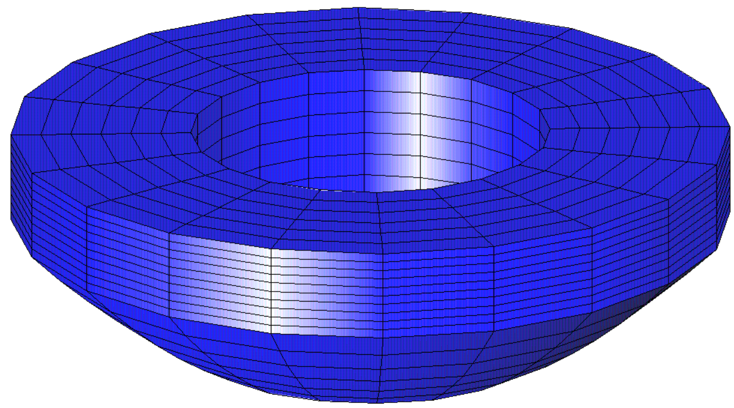
13.1.1 Quadratic depth profile

The quadratic depth function is defined by a quadratic polynomial:

$$z(x_0, x_1) = a_{00} + a_{10}x_0 + a_{01}x_1 + a_{20}x_0^2 + a_{11}x_0x_1 + a_{02}x_1^2$$

13.2 Examples

```
Annulus
exit
depth mapping
  quadratic depth
    -.75 0 0 .5 0. .5
exit
```



A DepthMapping starting from an Annulus. The depth is defined by a parabola.

13.3 Constructor

DepthMapping()

Description: Define a 3D Mapping from 2D Mapping by extending in the z-direction by a variable amount

13.4 setDepthFunction

int
setDepthFunction(Mapping & depth_)

Description: Supply a mapping that will define the depth.

depth_ (input) : Use this mapping for the depth, $z = depth(x_0, x_1)$.

13.5 setDepthFunction

int
setDepthFunctionParameters(real a0, real b0, real a1, real b1)

Description: Define the scaling parameters for the depth function.

a0,b0,a1,b1_ (input) :

13.6 setSurface

int
setSurface(Mapping & surface_)

Description: Supply a 2D mapping that will define the surface of the 3D domain.

surface_ (input) : 2D Mapping.

13.7 setQuadraticParameters

```
int
setQuadraticParameters(const real & a00_,
                       const real & a10_,
                       const real & a01_,
                       const real & a20_,
                       const real & a11_,
                       const real & a02_)
```

Description: Specify the parameters for a quadratic depth function:

$$z(x_0, x_1) = a_{00} + a_{10}x_0 + a_{01}x_1 + a_{20}x_0^2 + a_{11}x_0x_1 + a_{02}x_1^2$$

a00_, a10_,... (input): parameters in above formula.

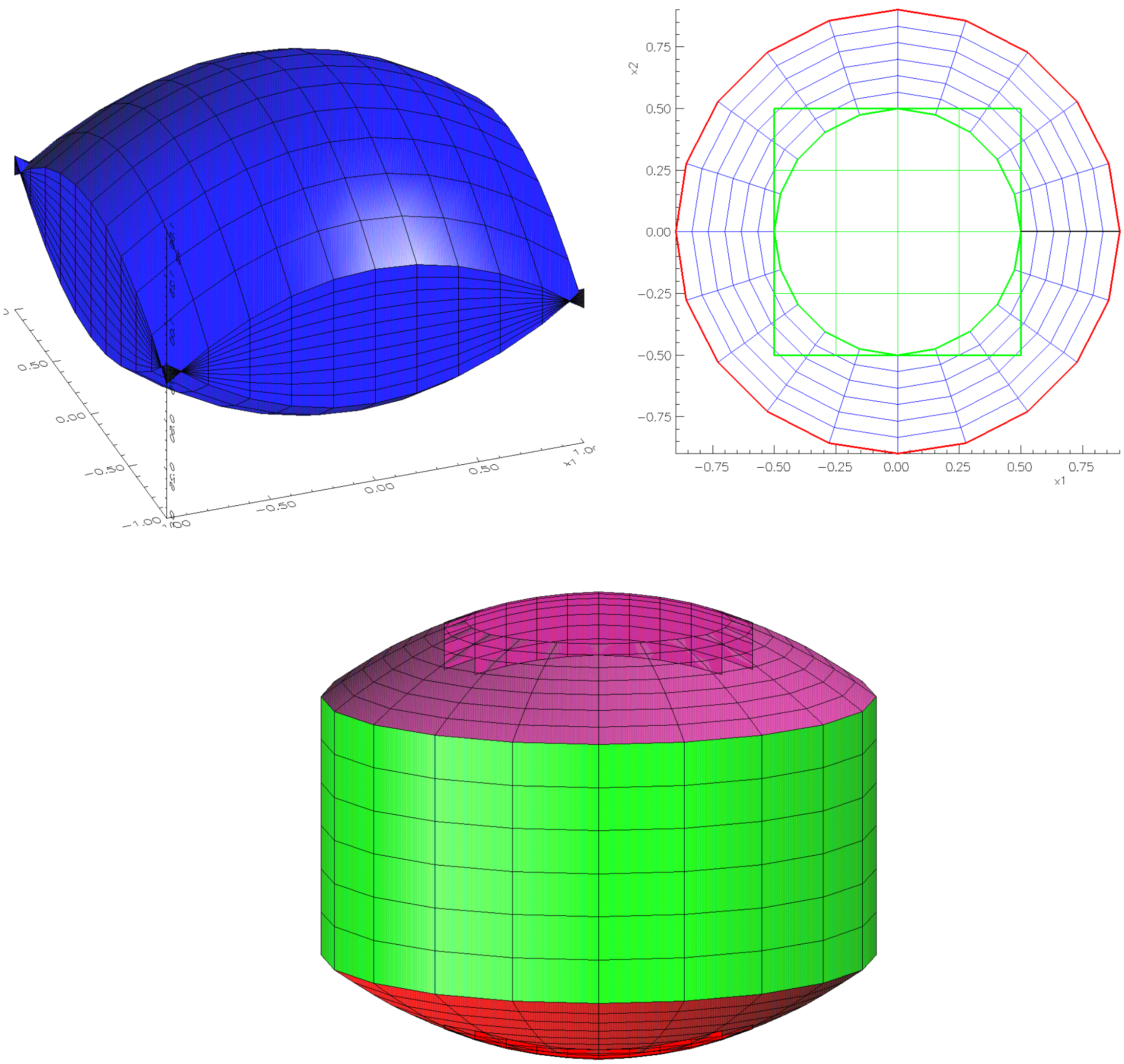


Figure 7: The DepthMapping (see bottom figure) is used to give a vertical dimension to mappings defined in the plane, `depth.cmd`. In this case a separate TFI mapping, top left, defines the vertical height function Both an annulus and a square (top right) are given a depth.

14 EllipticTransform

This Mapping was originally created by Eugene Sy. Changes made by Bill Henshaw.

**** This documentation is out of date as I have made lots of changes ****

14.1 Introduction

The program EllipticTransform.C performs smoothing on a desired mapping by solving a Poisson equation on the domain. A new elliptic mapping is created from the original mapping which is supplied as an initial condition.

The original mapping $\mathbf{M}$ maps the unit square U into physical space Ω_p. With the elliptic transform, a one to one function E' taking U into Ω_p is first created. Then, $\mathbf{M}^{-1}$ is applied to the solution in Ω_p to map the points back into the unit square. In this way, a mapping $\mathbf{D} = \mathbf{M}^{-1}(E')$ is obtained, where $\mathbf{D} : U \mapsto U$, and $\mathbf{D}$ is a data point mapping. The elliptic mapping $\mathbf{E}$ is then given by a composition, where

$$\mathbf{E}(U) = \mathbf{M}(\mathbf{D}(U))$$

14.2 The Governing Equations

The focus from here on will be on the creation of the mapping $\mathbf{E}$. Assume that U has coordinate directions ξ and η. The equations to be solved are:

$$\begin{aligned}\nabla^2 \xi &= P(\xi,\eta)\\ \nabla^2 \eta &= Q(\xi,\eta)\end{aligned}$$

These are transformed to computational space and take the form:

$$\begin{aligned}\alpha(\frac{\partial^2 x}{\partial \xi^2} + P\frac{\partial x}{\partial \xi}) + \beta(\frac{\partial^2 x}{\partial \eta^2} + Q\frac{\partial x}{\partial \eta}) - 2\gamma\frac{\partial^2 x}{\partial \xi \partial \eta} &= 0\\ \alpha(\frac{\partial^2 y}{\partial \xi^2} + P\frac{\partial y}{\partial \xi}) + \beta(\frac{\partial^2 y}{\partial \eta^2} + Q\frac{\partial y}{\partial \eta}) - 2\gamma\frac{\partial^2 y}{\partial \xi \partial \eta} &= 0\end{aligned}$$

$$\begin{aligned}\alpha &= (\frac{\partial x}{\partial \eta})^2 + (\frac{\partial y}{\partial \eta})^2\\ \beta &= (\frac{\partial x}{\partial \xi})^2 + (\frac{\partial y}{\partial \xi})^2\\ \gamma &= \frac{\partial x}{\partial \xi}\frac{\partial x}{\partial \eta} + \frac{\partial y}{\partial \xi}\frac{\partial y}{\partial \eta}\end{aligned}$$

These equations are solved on the unit square U using finite difference methods.

14.3 Control of the Boundary

The existing code has the ability to evaluate Dirichlet, orthogonal, and periodic boundary conditions for solving the above equations.

14.3.1 Dirichlet Conditions

If the boundary conditions are Dirichlet the locations of the boundary points must be correctly specified in the initial condition. These points are considered by the elliptic grid generator to be fixed, and computations are done only on the interior of the grid.

14.3.2 Orthogonal Boundary Conditions

If the user desires that the gridlines meet the boundary orthogonally, then the points on the boundary are given a degree of freedom, and are allowed to move along the boundary. For example, consider the boundary $\xi = 0$. Assume that the $(n+1)$st iteration is being computed, and let j represent the η coordinate. The following two equations are solved to give the location of the boundary points.

$$
\begin{aligned}
\mathbf{x}_\eta \cdot \mathbf{x}_\xi &= 0 \qquad (3)\\
\mathbf{x}^{n+1} &= \mathbf{x}^n + (\Delta\mathbf{x} \cdot \hat{\mathbf{x}}_\eta)\hat{\mathbf{x}}_\eta \qquad (4)
\end{aligned}
$$

Equation 1 is the orthogonality condition, and equation 2 prevents the boundary points from moving off the boundary. $\Delta\mathbf{x}$ represents the change in position of the point $\mathbf{x}$ dictated from equation 1. In other words, if $\mathbf{x}^n$ is the location of the point at the nth iteration step, equation 1 will steer the point towards a position $\mathbf{x}^*$ for the $(n+1)$st iterate. It follows that

$$
\begin{aligned}
\Delta\mathbf{x} &= \mathbf{x}^* - \mathbf{x}^n\\
\Delta\mathbf{x} \cdot \hat{\mathbf{x}}_\eta &= \mathbf{x}^* \cdot \hat{\mathbf{x}}_\eta - \mathbf{x}^n \cdot \hat{\mathbf{x}}_\eta
\end{aligned}
$$

To solve the equations, $\mathbf{x}_\xi$ in equation 1 is first forward differenced to obtain the following result.

$$
\begin{aligned}
{x^*}_{0,j} x_\eta &= y_\xi y_\eta + x_{1,j} x_\eta \qquad (5)\\
{y^*}_{0,j} y_\eta &= x_\xi x_\eta + y_{1,j} y_\eta \qquad (6)
\end{aligned}
$$

This gives an expression for $\mathbf{x}^* \cdot \mathbf{x}_\eta$, where $\mathbf{x}^*$ represents the position the boundary point wants to move to in order to satisfy the orthogonality condition. $\mathbf{x}_\eta$ and all the terms on the right hand side are calculated from the solution at the nth iterate. Normalizing by $\|\mathbf{x}_\eta\|$ yields the following:

$$
\begin{aligned}
{x^*}_{0,j} \hat{x}_\eta &= y_\xi \hat{y}_\eta + x_{1,j} \hat{x}_\eta \qquad (7)\\
{y^*}_{0,j} \hat{y}_\eta &= x_\xi \hat{x}_\eta + y_{1,j} \hat{y}_\eta \qquad (8)
\end{aligned}
$$

This is $\mathbf{x}^* \cdot \hat{\mathbf{x}}_\eta$. Then, since $\mathbf{x}^n \cdot \hat{\mathbf{x}}_\eta$ is also readily calculated from the nth iterate, the entire right hand side of equation 2 is known. This allows calculation of $\mathbf{x}^{n+1}$, and the continuation of the iteration.

Orthogonality at the boundary can also be obtained by a different means. As explained in Thompson, et al. [3], the boundary points can be fixed, and boundary orthogonality enforced by utilization of the proper forcing functions P and Q. In addition, the thickness of the boundary layer can be specified by the user.

Again, let $\xi = 0$, as above, and let j represent the η coordinate. Assume that the boundary layer thickness $\|\mathbf{x}_\xi\|$ is chosen, and calculate $\mathbf{x}_\eta$ and $\mathbf{x}_{\eta\eta}$ from the fixed locations of the boundary points. All this, together with the orthogonality condition

$$\mathbf{x}_\eta \cdot \mathbf{x}_\xi = 0$$

allows for the determination of $\mathbf{x}_\xi$. As shown in Knupp and Steinberg[2]. $\mathbf{x}_\xi$ is given by:

$$\mathbf{x}_\xi = \frac{\|\mathbf{x}_\xi\|}{\|\mathbf{x}_\eta\|}{\mathbf{x}_\eta}^\perp \qquad (9)$$

Here, ${\mathbf{x}_\eta}^\perp$ is the vector perpendicular to the vector $\mathbf{x}_\eta$. In addition to this, the method requires $\mathbf{x}_{\xi\xi}$, and this is calculated by means of the Pade approximation

$$\mathbf{x}_{\xi\xi}|_0 = \frac{-7\mathbf{x}_{1,j} + 8\mathbf{x}_{1,j} - \mathbf{x}_{3,j}}{2\Delta\eta^2} - 3\frac{\mathbf{x}_\xi|_0}{\Delta\eta} \qquad (10)$$

Using this information, the proper forcing functions P and Q can be determined.

$$
\begin{aligned}
P(\xi,\eta) &= -\frac{\mathbf{x}_\xi \cdot \mathbf{x}_{\xi\xi}}{\|\mathbf{x}_\xi\|^2} - \frac{\mathbf{x}_\xi \cdot \mathbf{x}_{\eta\eta}}{\|\mathbf{x}_\eta\|^2}\\
Q(\xi,\eta) &= -\frac{\mathbf{x}_\eta \cdot \mathbf{x}_{\eta\eta}}{\|\mathbf{x}_\eta\|^2} - \frac{\mathbf{x}_\eta \cdot \mathbf{x}_{\xi\xi}}{\|\mathbf{x}_\xi\|^2}
\end{aligned}
$$

Once the appropriate P and Q are determined for the boundary, the values are interpolated onto the interior points using a linear scaling, and the iteration is continued.

Note that because second order differences are being lagged, heavy underrelaxation is required for this scheme to converge (Knupp and Steinberg [2]).

14.3.3 Periodic Boundaries

Two types of periodic boundaries exist. The first is derivative periodic and the second is function periodic. Derivative periodicity involves identical derivatives on the boundary, but not necessarily identical positions. Function periodicity involves matching both the derivatives and the positions at the boundary points (as in the case of an annulus).

In either case, the values beyond the boundary are evaluated by means of ghost points. A ghost array allows for calculation of values on the boundary in much the same way they are found on the interior.

14.4 Sources

Should clustering of points or lines be necessary in the interior, certain points or lines may be designated as being lines of attraction. This involves manipulation of the source terms P and Q before iteration begins. If a coordinate line is to be made a line of attraction, the attraction power π must be specified along with the diffusivity δ. With these two parameters, the following expressions for P and Q are evaluated at all points in the field.

$$
\begin{aligned}
P_{line}(\xi,\eta) &= -\sum_{i=1}^{N} \pi_i Sign(\xi-\xi_i) e^{-\delta_i|\xi-\xi_i|} \\
Q_{line}(\xi,\eta) &= -\sum_{j=1}^{M} \pi_j Sign(\eta-\eta_j) e^{-\delta_j|\eta-\eta_j|}
\end{aligned}
$$

Here, N and M represent the number of ξ and η lines of attraction respectively. Should points of attraction be desired, power and diffusivity are specified for each point, and two more sums are evaluated.

$$
\begin{aligned}
P_{point}(\xi,\eta) &= -\sum_{i=1}^{L} \pi_i Sign(\xi-\xi_i) e^{-\delta_i|\xi-\xi_i|} \\
Q_{point}(\xi,\eta) &= -\sum_{j=1}^{L} \pi_j Sign(\eta-\eta_j) e^{-\delta_j|\eta-\eta_j|}
\end{aligned}
$$

Here, L is the number of point sources. Note that a source can be made into a sink by merely changing the sign in front of the power π.

14.5 Using the Elliptic Grid Generator With Ogen

The user is assumed to be familiar with generation of mappings in Ogen. A mapping must first be made as an initial condition for the elliptic smoother.

14.5.1 Grid Dimensions

The first thing to specify after choosing a mapping is the amount of grid refinement desired. The number of grid lines in i and j (or ξ and η respectively) should be entered before all else, as these parameters are needed for correct implementation of the boundary conditions.

14.5.2 Boundary Conditions

The appropriate GRID boundary conditions should be entered next. These are not to be confused with the boundary conditions for the physical problem to be solved later, and the switches are completely independent. The following choices are available:

- -1: Refers to a **periodic** boundary condition. This is selected by default if a periodic boundary is declared when the original mapping is made. If a periodic boundary is not specified when the original mapping was created, this boundary condition should not be used.

- 1: Refers to a **Dirichlet** boundary condition. The positions of the boundary points in the original mapping are used as the boundary condition for the elliptic map.

- 2: Refers to a **orthogonal** boundary condition. This forces the gridlines to meet the boundary in an orthogonal fashion. The boundary points are free to move, but only along the boundary.

- 3: Refers to the **combined** boundary condition. The boundary points are fixed as in the Dirichlet case, but the sources and sinks are modified so as to guarantee orthogonality at the boundary. This boundary condition requires that the user specify the thickness of the boundary layer.

The boundary conditions are stored in a 2-dimensional array **gridBc(i,j)**, where i and j range from 0 to 1. On the unit square, the following are the locations of the boundaries:

- gridBc(0,0): Refers to the boundary condition on $0 \leq x \leq 1, y = 0$.
- gridBc(0,1): Refers to the boundary condition on $x = 0, 0 \leq y \leq 1$.
- gridBc(1,0): Refers to the boundary condition on $0 \leq x \leq 1, y = 1$.
- gridBc(1,1): Refers to the boundary condition on $x = 1, 0 \leq y \leq 1$.

14.5.3 Sources and Sinks

If the user decides that lines or points of attraction (repulsion) are desired for the elliptic grid, these can be specified. The selection *Poisson i-line sources* creates lines of constant ξ into lines of attraction. Similarly, *Poisson j-line sources* turns lines of constant η into lines of attraction, and *Poisson point sources* creates points of attraction in $\xi - \eta$ space.

The power and diffusivity of each source must be selected carefully. Too much power or too little diffusivity can mar convergence of the grid.

14.5.4 Other Functions

There are several other switches that can be used. These are:

- *change SOR parameter*: This changes the value of ω used for either overrelaxation or underrelaxation. Setting $\omega = 1$ indicates a point Gauss- Seidel method, and is a good conservative first choice. Note that if **combined** boundary conditions are used, then ω needs to be quite small, usually on the order of 0.05 or 0.1. This is because of the lagged second order quantities that the scheme requires.
- *set maximum number of iterations*: This controls the maximum amount of iterations for the grid to converge.
- *set epsilon for convergence*: This value is the indicator for the convergence of the elliptic grid. If the differences between successive iterations drops below epsilon, then the grid is said to have converged. This is preset to 10^{-5}.
- *set number of periods*: If the grid is periodic (either derivative or function), then any sources or sinks in the field need to be made periodic as well. The number entered here indicates the number of periods desired for these sources and sinks. 1 is the default value. Increasing this number is crucial if strong source terms exist, and 5 or 7 may be needed to properly resolve the periodicity.

14.6 In Conclusion

The elliptic transform is a fine way to smooth out a grid, and frees the user from the restriction to simple geometries. Unfortunately, convergence of the routine is not guaranteed for all geometries, and for all powers of sources and sinks.

The routine, however, does provide an ability to deal with boundaries well, and can greatly simplify many complex computations.

14.7 Member functions

14.7.1 Constructor

EllipticTransform()

Purpose: Create a mapping that can be used to generate an elliptic grid from an existing grid. This can be useful to smooth out an existing Mapping.

14.7.2 get

int
get(const GenericDataBase & dir, const aString & name)

Description: Get a mapping from the database.

dir (input): get the Mapping from a sub-directory of this directory.

name (input) : name of the sub-directory to look for the Mapping in.

14.7.3 put

int
put(GenericDataBase & dir, const aString & name) const

Description: Save a mapping into a database.

dir (input): put the Mapping into a sub-directory of this directory.

name (input) : name of the sub-directory to save the Mapping in.

14.7.4 generateGrid

void
generateGrid(GenericGraphicsInterface *gi = NULL,
GraphicsParameters & parameters =Overture::nullMappingParameters())

Description: This function performs the iterations to solve the elliptic grid equations.

gi (input) : supply a graphics interface if you want to see the grid as it is being computed.

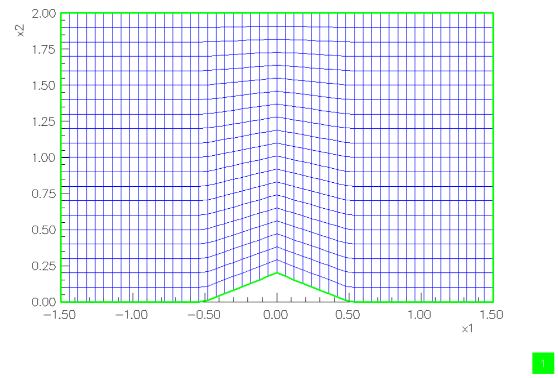
parameters (input) : optional parameters used by the graphics interface.

14.8 Examples

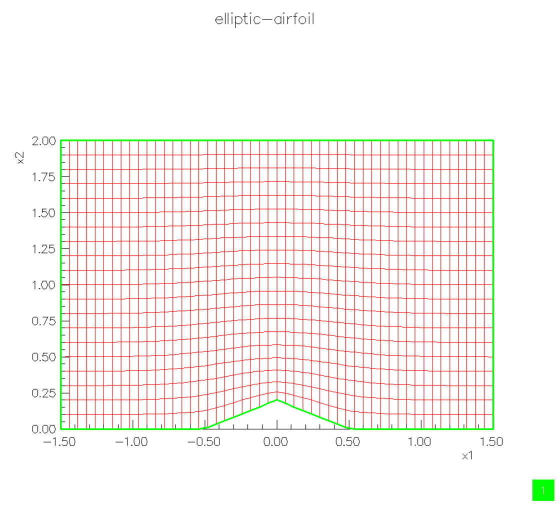
14.8.1 Smoothed out diamond airfoil

In the left column is the command file that was used to generate the grid on the bottom right.

```
*
* Smooth a diamond airfoil with the
*  elliptic transform mapping.
*
* first make a diamond airfoil
*
Airfoil
  airfoil type
    diamond
  thickness-chord ratio
    .2
  lines
    51 21
exit
*
* now smooth the diamond airfoil
*
elliptic
  *
  * do not project back onto the original mapping
  * since it has a discontinuity
  *
  project onto original mapping (toggle)
  *
  * now generate the elliptic transform:
  *
  elliptic smoothing
exit
```



Diamond airfoil before elliptic transform.



Diamond airfoil after elliptic transform.

15 FilletMapping

This mapping can be used to create a fillet grid or a collar grid in order to join together two intersecting surfaces. A fillet grid smooths out the intersection while the collar grid does not.

This mapping will automatically compute the fillet given two intersecting surfaces. Various parameters control the resulting surface:

orientation: There are 4 possible quadrants in which to place the fillet.

width: This distance defines the width over which the fillet blends between the two surfaces and thus determines how rounded the fillet is. A width of zero will result in a collar grid which has a corner in it.

overlapWidth: Determines the distance to which the fillet extends onto each surface once it has touched the surface.

15.1 Description of Approach

Here are the basic steps that are used to create a fillet or collar grid:

intersect surfaces: Given two intersecting surfaces we first compute the curves(s) of intersection using the `IntersectionMapping`. (For a 2D fillet grid the intersection curves are just the points of intersection.

generate surface grids: The next step is to generate a hyperbolic surface grid on each of the two surfaces, using the curve of intersection as a starting curve (not necessary to do in 2D). The surface grid is grown in both directions from the starting curve. The `HyperbolicSurfaceMapping` is used to generate these grids.

blend surface grids: The fillet grid is defined as a blending of the two surface grids. The precise description of this blending is given below.

15.2 Fillet for two intersecting surfaces

To define a fillet to join two intersecting surfaces, $\mathbf{S}_1$, $\mathbf{S}_2$ we use

$$\begin{aligned}
\mathbf{c}_I(r_1) &= \text{Curve of intersection} \\
\mathbf{c}_1(r_1,r_2) &= \text{Grid on surface 1, with } \mathbf{c}_1(r_1,.5) = \mathbf{c}_I(r_1) \\
\mathbf{c}_2(r_1,r_2) &= \text{Grid on surface 2, with } \mathbf{c}_2(r_1,.5) = \mathbf{c}_I(r_1)
\end{aligned}$$

If the parameter r_1 is tangential to the intersection and r_2 varies in the direction normal to the intersection then the fillet is defined by blending the two surface grids:

$$\begin{aligned}
\mathbf{x} &= b(s)\mathbf{c}_1(r_1,s_1(r_2)) + (1-b(s))\mathbf{c}(r_1,s_2(r_2)) \\
b &= \frac{1}{2}(1+\tanh(\beta(r_2-.5)))
\end{aligned}$$

where the parameter variables $s_i(r_2)$ are chosen to be quadratic polynomials in r_2,

$$s_i = a_{i0}(r_1) + r_2(a_{i1}(r_1) + r_2 a_{i2}(r_1))$$

where

$$\begin{aligned}
c_{i,0} &= .5 && \text{intersection point} \\
c_{i,1} &= c_{i,0} - pm[i] * .5 * filletWidth/crNorm && \text{distance from intersection point for c1} \\
c_{i,2} &= c_{i,0} - pm[i] * (.5 * filletWidth + filletOverlap)/crNorm && \text{distance from intersection point for c2} \\
c_{i,3} &= c_{i,0} + pm[i] * shift * .5 * filletWidth/crNorm \\
a_{i0} &= c_{i,2+i} \\
a_{i1} &= c_{i,3-i} - c_{i,2+i} + (16./3.) * (c_{i,1} - .75 * c_{i,2} - .25 * c_{i,3}) \\
a_{i2} &= -((16./3.) * (c_{i,1} - .75 * c_{i,2} - .25 * c_{i,3}))
\end{aligned}$$

15.3 setCurves

int
setCurves(Mapping & curve1,
Mapping & curve2)

Description: Supply the curves or surfaces from which the fillet will be defined.

curve1, curve2 (input):

15.4 map

void
map(const realArray & r, realArray & x, realArray & xr, MappingParameters & params)

Purpose: Evaluate the TFI and/or derivatives.

15.5 update

int
update(MappingInformation & mapInfo)

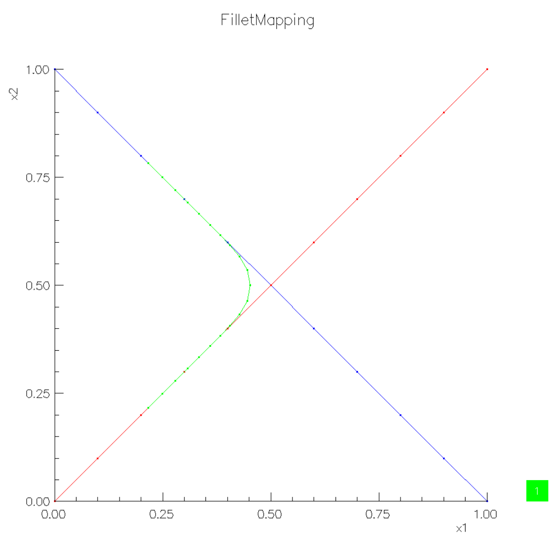
Purpose: Interactively create and/or change the Fillet mapping.

mapInfo (input): Holds a graphics interface to use.

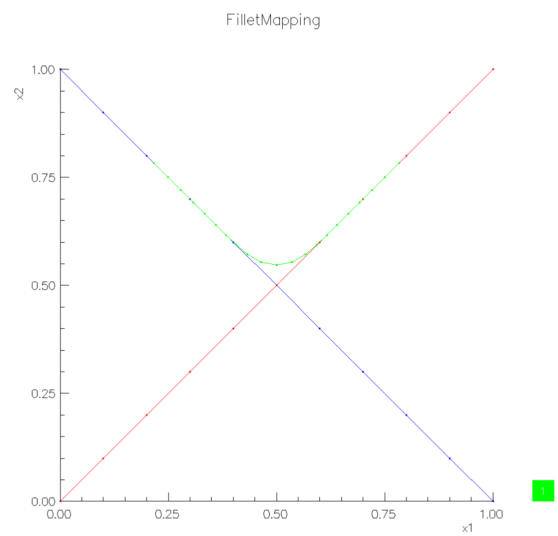
15.6 examples

15.6.1 2D Fillet joining two lines

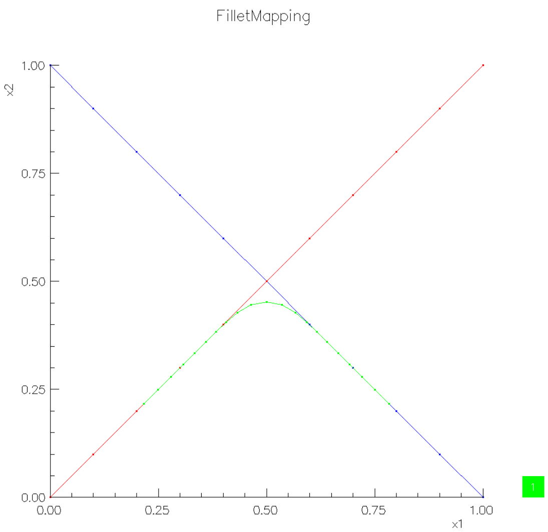
This is a 2D example showing a fillet that joins two line segments. These figures show the four possible fillets that can be generated between intersecting curves (or surfaces).



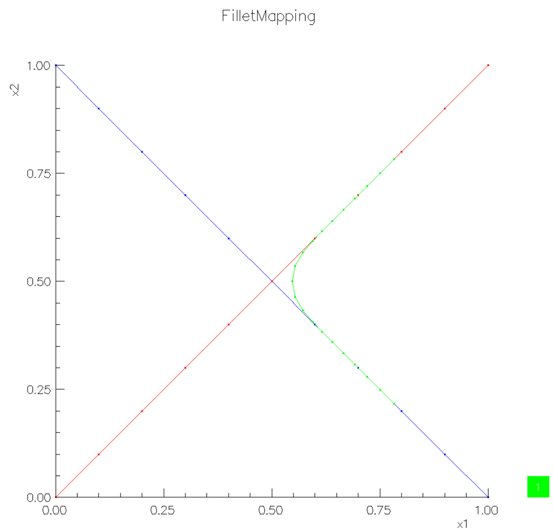
A fillet grid joining two lines. The orientation is `curve 1- to curve 2-`.



A fillet grid joining two lines. The orientation is `curve 1+ to curve 2-`.



A fillet grid joining two lines. The orientation is `curve 1- to curve 2+`.

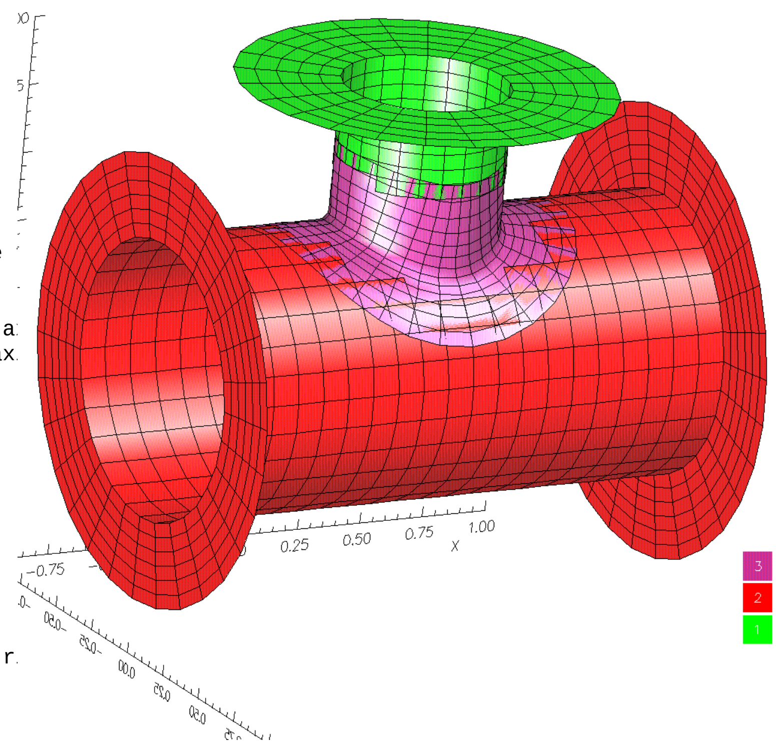


A fillet grid joining two lines. The orientation is `curve 1+ to curve 2+`.

15.6.2 Fillet to join two cylinders

In the left column is the command file that was used to generate the grid on the right.

```
  Cylinder
    orientation
      1 2 0
    bounds on the axial variable
      -1. 1.
    bounds on the radial variable
      .5 .75
    boundary conditions
      -1 -1 1 2 3 0
    mappingName
      main-cylinder
    lines
      31 21 6
    exit
  Cylinder
    mappingName
      top-cylinder
    orientation
      2 0 1
    bounds on the axial variable
      .25 1.
    bounds on the radial variable
      .3 .6
    boundary conditions
      -1 -1 0 2 3 0
    lines
      25 15 5
    exit
*
  fillet
   * define more lines for computing the
   lines
     81 41 41 21
    Start curve 1:main-cylinder (side=0,a
    Start curve 2:top-cylinder (side=0,ax
    orient curve 1+ to curve 2-
    compute fillet
*    pause
  exit
* build a volume grid around the fillet
  hyperbolic
    grow grid in opposite direction
    distance to march .2
    points on initial curve 31 12
    lines to march 7
    uniform dissipation .1
    outward splay .25 .25 .25 .25 (left,r
    * show parameters
    BC: bottom outward splay
    BC: top outward splay
    generate

    lines
      31 12 6
    mappingName
      cylinderFillet
    share
      0 0 0 0 0 0

    exit




   choose curves
     main-cylinder (side=0,axis=2)
     top-cylinder (side=0,axis=2)
   orient curve 1- to curve 2+
   compute
   * reduce the lines for actual fillet
*   lines
*     31 12
```

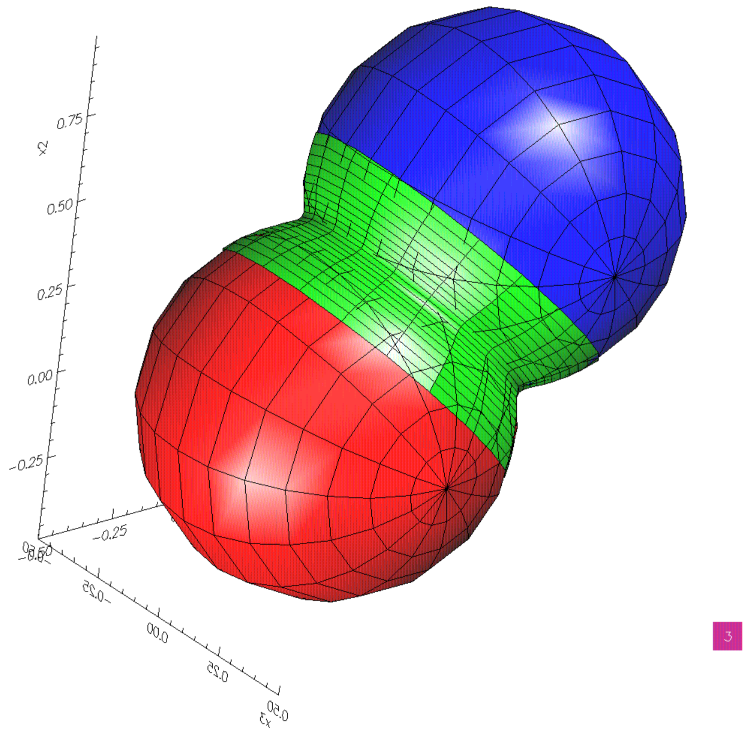


A fillet grid joining two cylinders. The fillet is created with the aid of hyperbolic grid generation.

15.6.3 Fillet to join two spheres

In the left column is the command file that was used to generate the grid on the right.

```
* build a sphere
  Sphere
    surface or volume (toggle)
    mappingName
      sphere1
  exit
* build a second sphere
  Sphere
    surface or volume (toggle)
    mappingName
      sphere2
    centre for sphere
      .5 .5
    exit
* build the fillet
  fillet
    Start curve 1:sphere1
    Start curve 2:sphere2
    orient curve 1+ to curve 2-
    compute fillet
    choose curves
    sphere1
    sphere2
   orient curve 1+ to curve 2-
    compute fillet
```



A fillet grid (green) joining two spheres.

15.7 HyperbolicMapping

See the separate document *The Overture Hyperbolic Grid Generator* [1].

16 IntersectionMapping

This mapping class can compute the intersection between two other mappings, such as the curve of intersection between two surfaces. See the comments with the `determineIntersection` function for a description of the fairly robust way in which we find the intersection.

16.1 Constructor

IntersectionMapping()

Purpose: Default Constructor

Author: WDH

16.2 Constructor

IntersectionMapping(Mapping & map1_,
Mapping & map2_)

Purpose: Define a mapping for the intersection of map1_ and map2_

map1_, map2_ : two surfaces in 3D

16.3 intersect

int
intersect(Mapping & map1_, Mapping & map2_,
GenericGraphicsInterface *gi =NULL,
GraphicsParameters & params =nullGraphicsParameters)

Description: Determine the intersection between two mappings, optionally supply graphic parameters so the intersection curves can be plotted, (for debugging purposes). NEW FEATURE: If the intersection curve has disjoint segments, these segments will be stored as sub curves in the NURBS for the physical and parameter curves on each surface.

map1_, map2_ (input) : These two mappings will be intersected.

gi, paramas (input) : Optional parameters for graphics.

Return value: 0 for success

16.4 intersect

int
intersectWithCompositeSurface(Mapping & map1_, CompositeSurface & cs,
GenericGraphicsInterface *gi =NULL,
GraphicsParameters & params =nullGraphicsParameters)

Description: A Protected routine that computes the intersection between a Mapping and a CompositeSurface.

map1_, map2_ (input) : These two mappings will be intersected.

gi, paramas (input) : Optional parameters for graphics.

Return value: 0 for success

Output: The output intersection curve is a NurbsMapping. The number of subcurves of this mapping defines the number of disconnected components of the intersection.

16.5 newtonIntersection

int
newtonIntersection(realArray & x, realArray & r1, realArray & r2, const realArray & n)

Description: This is a protected routine to determine the exact intersection point on two surfaces using Newton's method.

Solve for (x,r1,r2) such that

```
map1(r1) - x = 0
map2(r2) - x = 0
n.x = c
```

x(.,3) (input/output) : initial guess to the intersection point (in the Range space)

r1(.,2) (input/output): initial guess to the intersection point (in the domain space of map1)

r2(.,2) (input/output): initial guess to the intersection point (in the domain space of map2)

n(.,3) : a normal vector to a plane that crosses the intersection curve, often choosen to be n(i,.) = x(i+1,.) - x(i-1,.) if we are computing x(i,.)

Return values: 0 for success. 1 if the newton iteration did not converge, 2 if there is a zero normal vector.

16.6 project

int
project(realArray & x,
int & iStart,
int & iEnd,
periodicType periodic)

Description: Project the points x(iStart:iEnd,0:6) onto the intersection NOTE: When the points are projected onto the curves it is possible that points fold back on themselves if they get out of order. This routine will try and detect this situation and it may remove some points to fix the problem. Return values: 0 for success, otherwise failure.

16.7 determineIntersection

int
determineIntersection(GenericGraphicsInterface *gi =NULL,
GraphicsParameters & params =nullGraphicsParameters)

Description: This is a protected routine to determine the intersection curve(s) between two surfaces.

Notes: (1) First obtain an initial guess to the intersection: Using the bounding boxes that cover the surface to determine a list of pairs of (leaf) bounding boxes that intersect. Triangulate the surface quadrilaterals that are found in this "collision" list and find all line segments that are formed when two triangles intersect.

(2) Join the line segments found in step 1 into a continuous curve(s). There will be three different intersection curves – a curve in the Range space (x) and a curve in each of the domain spaces (r). Since the domain spaces may be periodic it may be necessary to shift parts of the domain-space curves by +1 or -1 so that the curves are continuous. Note that the domain curves will sometimes have to be outside the unit square. It is up to ?? to map these values back to [0,1] if they are used.

(3) Now fit a NURBS curve to all of the intersection curves, using chord-length of the space-curve to parameterize the three curves. (4) Re-evaluate the points on the curve using Newton's method to obtain the points that are exactly on on the intersection of the surfaces. Refit the NURBS curves using these new points.

Return values: 0 for success, otherwise failure.

16.8 map

int
**reparameterize(const real & arcLengthWeight =1.,
const real & curvatureWeight =.2)**

Purpose: Redistribute points on the intersection curve to place more points where the curvature is large.

Description: The default distribution of points in the intersection curve is equally spaced in arc length (really chord length). To cluster more points near sharp corners, call this routine with a non-zero value for `curvatureWeight`. In this case the points will be placed to equidistribute the weight function

```
   w(r) = 1 + arcLength(r)*arcLengthWeight + curvature(r)*curvatureWeight
where
   arcLength(r) =  | x_r |
   curvature(r) =  | x_rr |    (*** this is not really the curvature, but close ***)
```

Note that the point distribution only depends on the ratio of arcLengthWeight to curvatureWeight and not on their absolute vaules. The weight function must be positive everywhere. Also note that for the unit circle, $|x_r| = 2\pi$ and $|x_{rr}| = (2\pi)^2$ so that the curvature is naturally 2π times larger in the weight function.

arcLengthWeight (input) : weight for the arc length, should be positive.

curvatureWeight (input) : weight for the curvature, should normally be non-negative.

16.9 intersectCurves

int
**intersectCurves(Mapping & curve1,
Mapping & curve2,
int & numberOfIntersectionPoints,
realArray & r1,
realArray & r2,
realArray & x)**

Description: Determine the intersection between two 2D curves.

curve1, curve2 (input) : intersect these curves

numberOfIntersectionPoints (output): the number of intersection points found.

r1,r2,x (output) : r1(i),r2(i),x(0:1,i) the intersection point(s) for $i = 0, \ldots, numberOfIntersectionPoints - 1$ are $curve1(r1(i)) = curve2(r2(i)) = x(i)$

16.10 map

void
map(const realArray & r, realArray & x, realArray & xr, MappingParameters & params)

Purpose: Evaluate the intersection curve.

16.11 get

int
get(const GenericDataBase & dir, const aString & name)

Purpose: get a mapping from the database.

16.12 put

int
put(GenericDataBase & dir, const aString & name) const

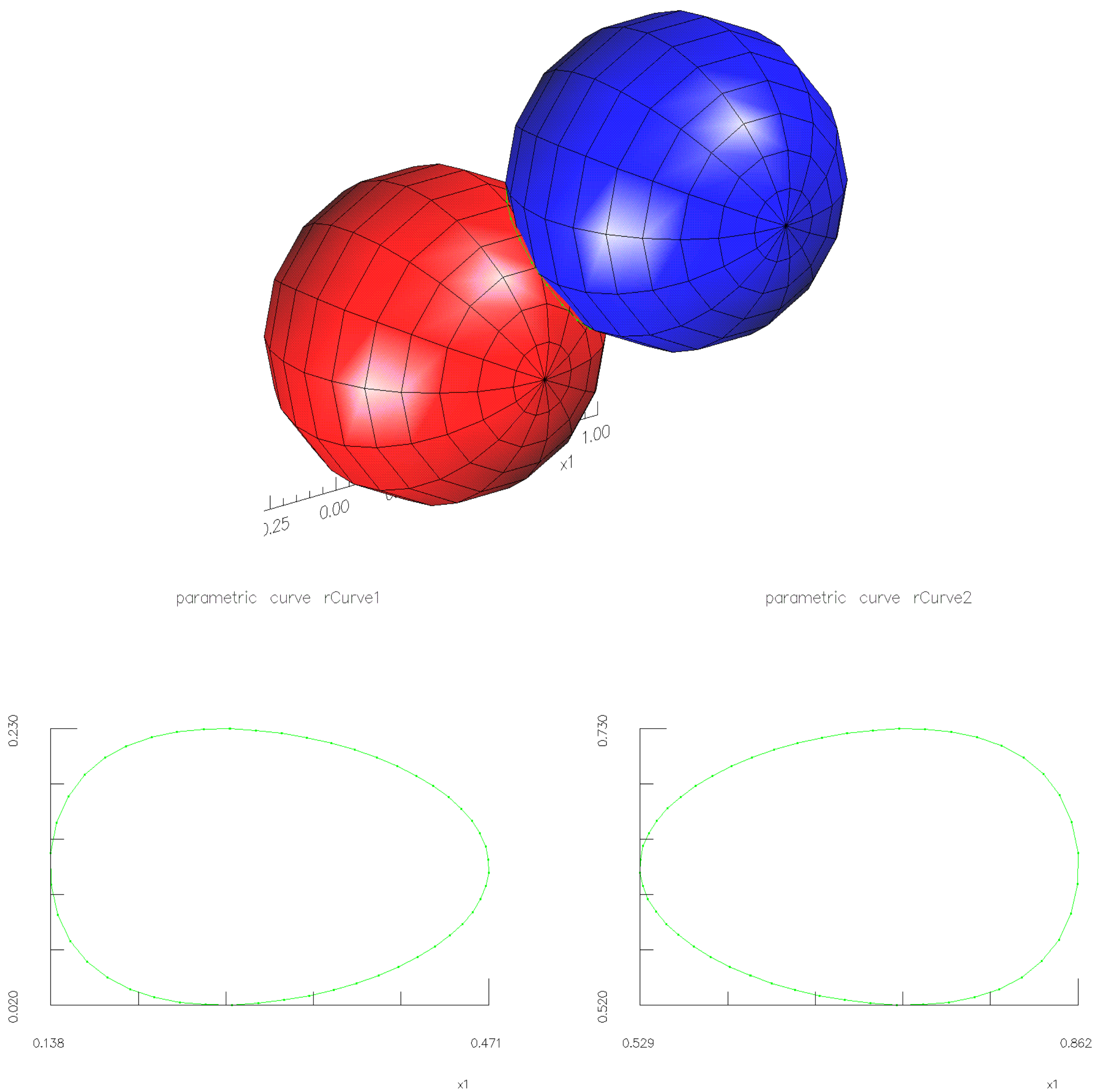
Purpose: put the mapping to the database.

16.13 update

int
update(MappingInformation & mapInfo)

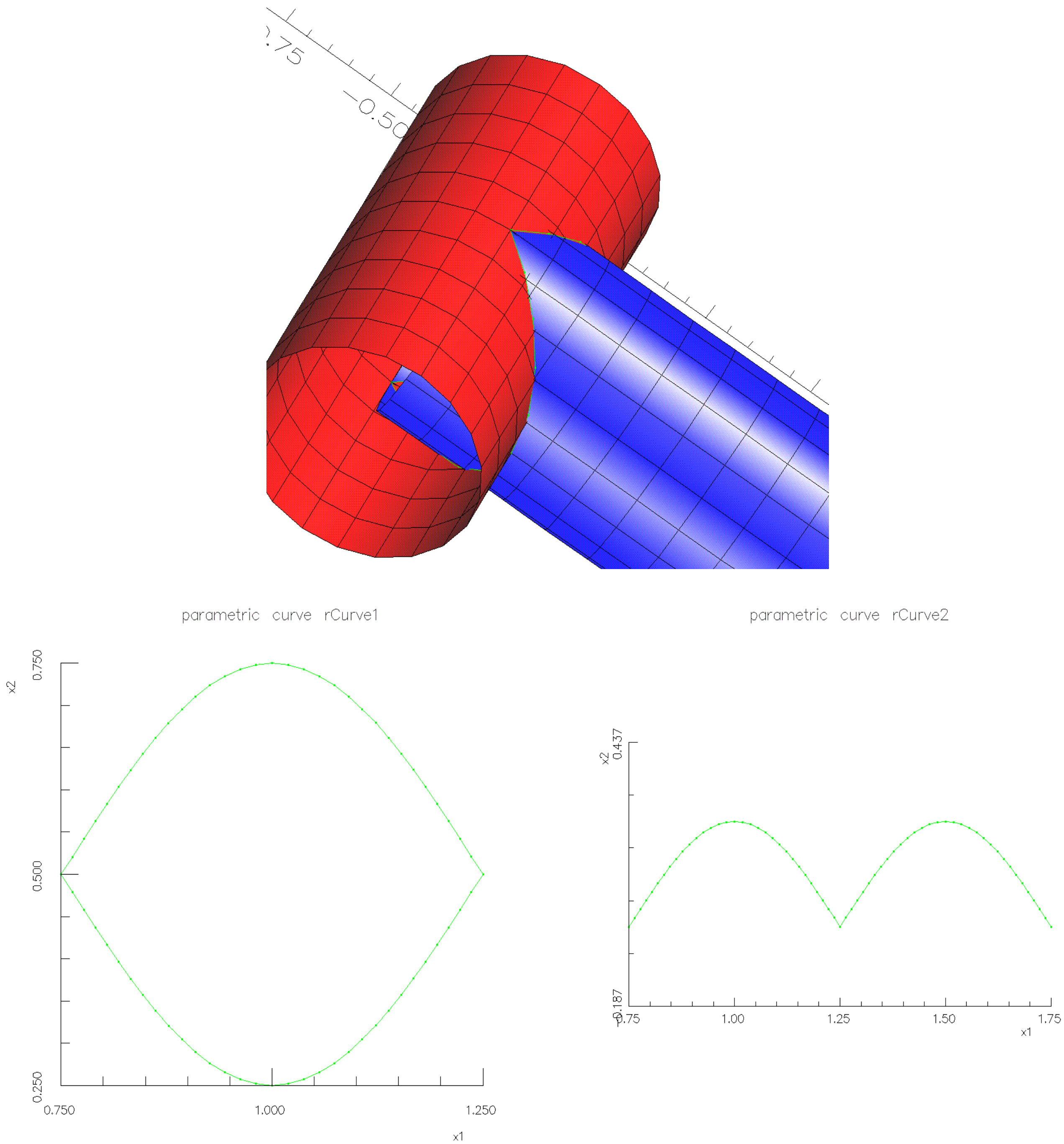
Purpose: Interactively create and/or change the mapping.

mapInfo (input): Holds a graphics interface to use.



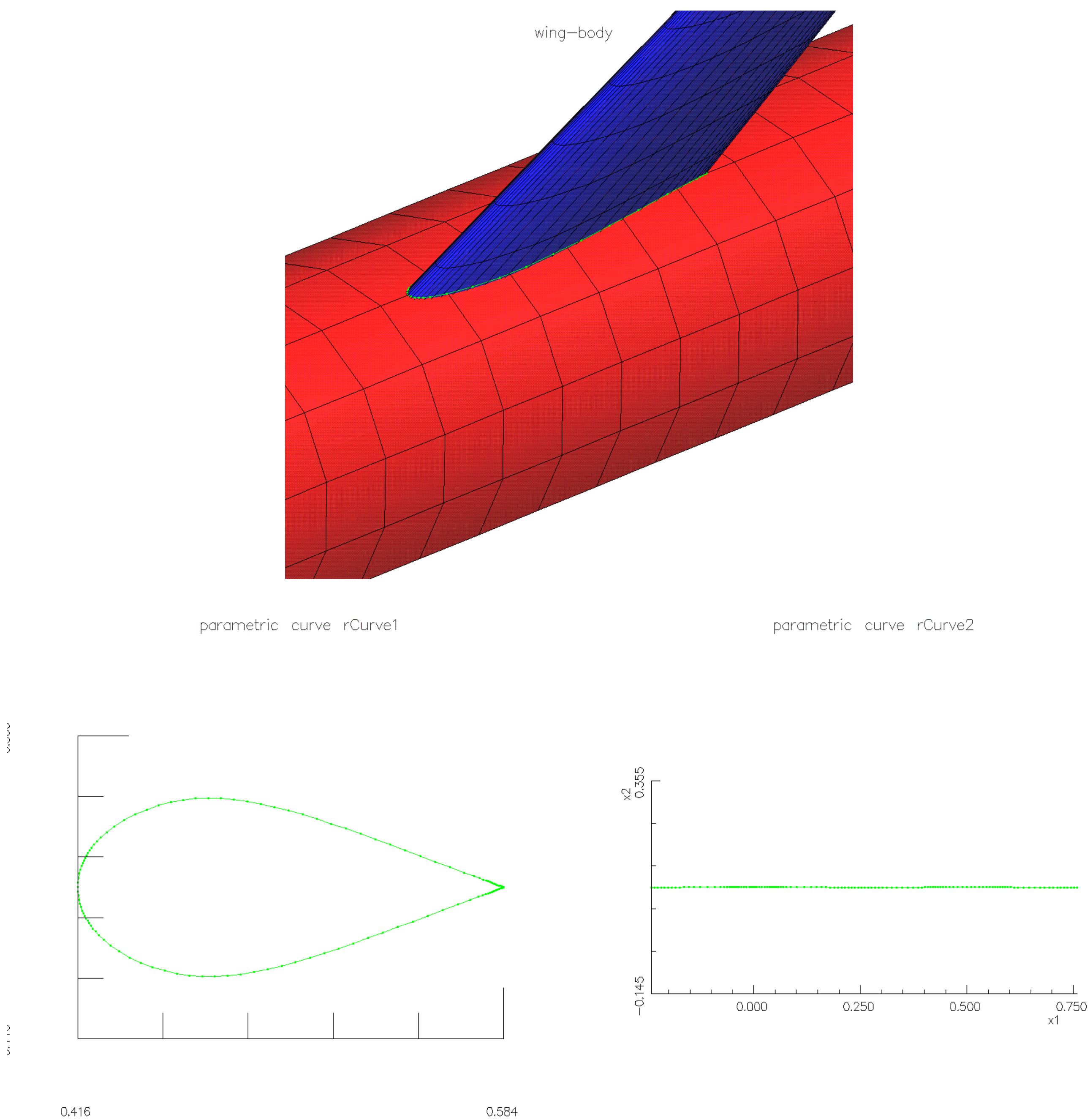
(a) (b)

Figure 8: The sphere-sphere intersection curve in the range space and the domain spaces (unit squares).



(a) (b)

Figure 9: The cylinder-cylinder intersection curves in the range space and the domain spaces (unit squares). This is a reasonably hard case since the cylinders have the same radius and thus the surfaces are tangent at two points.



(a) (b)

Figure 10: Intersection curve for a wing-body configuration. The curve was reparameterized weighting arclength and curvature in order to redistribute more points to the high curvature regions.

17 JoinMapping

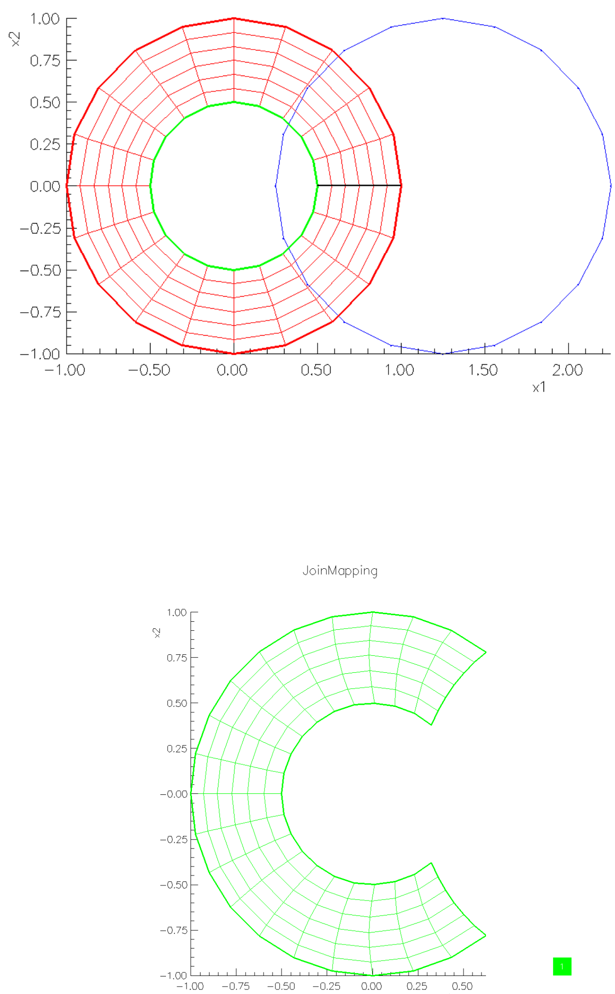
This mapping can be used to join together two Mappings that intersect. This is an alternative way to the `FilletMapping` to connect two intersecting surfaces.

The protypical example of the use of a `JoinMapping` is the intersection of a wing (the *intersector* mapping, i.e. the mapping that will be changed) with a fuselage (the *intersectee* mapping). If the end of the wing does not match exactly to the fuselage, there will be a part of of the wing that extends inside the fuselage. The `JoinMapping` can be used to remove the part of the wing that is inside the fuselage and reparameterize the rest of the wing so that the new wing matches exactly to the fuselage.

17.1 A 2D example

In this first example we consider an annulus that intersects a circle. We generate a new mapping that consists of the portion of the annulus that lies outside the circle.

```
* join an annulus to a circle
  Annulus
    exit
  Circle or ellipse
    specify centre
      1.25 0
    exit
  join
    choose curves
    Annulus
    circle
    compute join
```



A new mapping (bottom) is generated that replaces the annulus by a partial annulus that exactly matches to the circle.

17.2 Intersecting surfaces

Consider the case where the *intersector* and *intersectee* Mappings are both surfaces in 3D. The `JoinMapping` will first compute the curve of intersection between the *intersector* and the *intersectee* mappings. The curve of intersection, as generated by the `IntersectionMapping`, will have three representations:

- A space curve $\mathbf{x}_i(s)$, $s \in [0,1]$. matching the curve of intersection in physical space.
- A curve in parameter space of the intersector, $\mathbf{r}_i(s)$, $s \in [0,1]$, that is the pre-image of the space curve $\mathbf{x}_i(s)$. Thus if $\mathbf{x} = \mathbf{C}_i(\mathbf{r})$ denotes the intersector mapping, then $\mathbf{x}_i(s) = \mathbf{C}_i(\mathbf{r}_i(s))$.
- There is also a parametric curve for the intersectee mapping, $\mathbf{r}_e(s)$, with $\mathbf{x}_i(s) = \mathbf{C}_e(\mathbf{r}_i(s))$, where $\mathbf{C}_e(\mathbf{r})$ is the intersectee mapping.

To reparmeterize the intersector mapping we first define a new mapping in the parameter space of the intersector that is bounded on one side by the parametric intersection curve $\mathbf{r}_i$. In the typical case this new mapping can be defined by trans-finite interpolation (`TFIMapping`), such as

$$\mathbf{P}(\mathbf{r}) = (1 - r_2)\mathbf{r}_i(r_1) + r_2(r_1, 1)$$

In this case the curve $\mathbf{r}_i$ is assumed to be mainly in the r_1 direction and we have chosen to extend the patch to $r_2 = 1$.

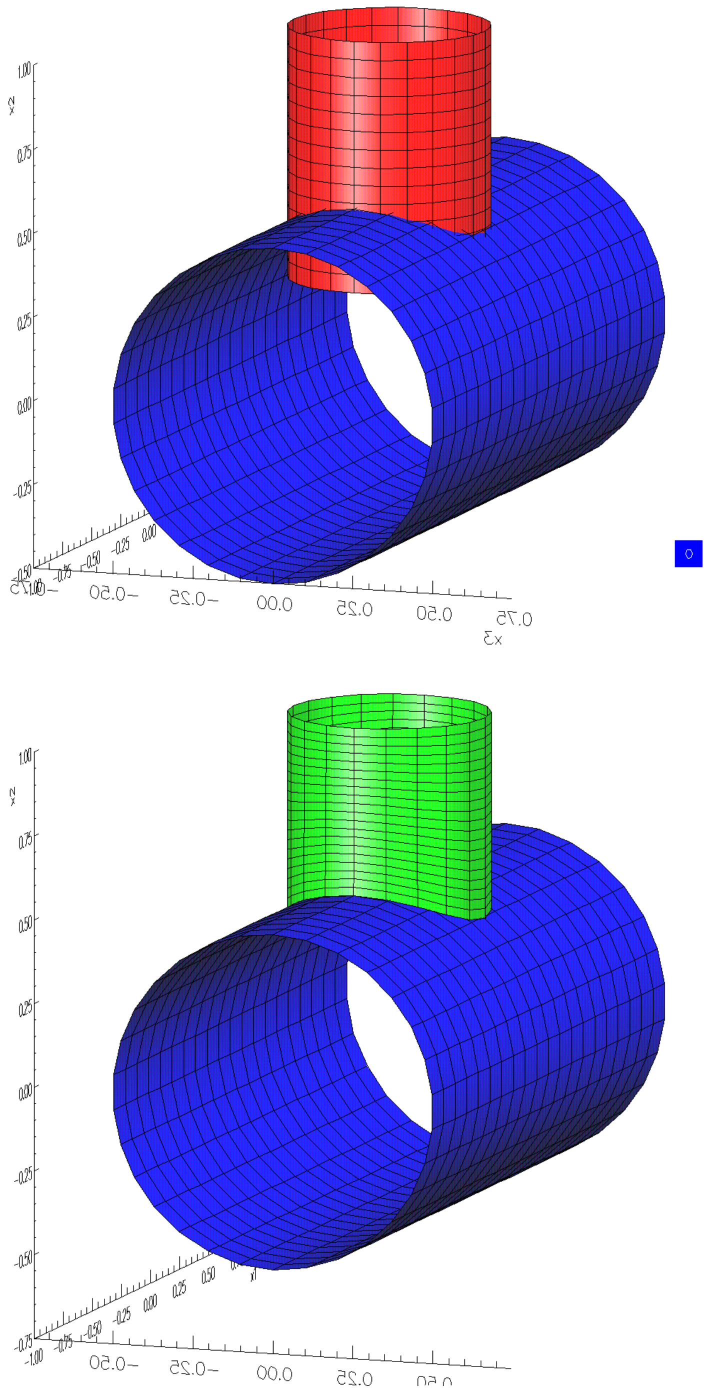
The `JoinMapping` is now defined by the composite Mapping,

$$\mathbf{J}(\mathbf{r}) = \mathbf{C}_i(\mathbf{P}(\mathbf{r}))$$

Through the definitions we see that $\mathbf{J}(\mathbf{r})$ will exactly match the curve of intersection at $r_2 = 0$,

$$\mathbf{J}(r_1, 0) = \mathbf{C}_i(\mathbf{P}(r_1, 0)) = \mathbf{C}_i(\mathbf{r}_i(r_1)) = \mathbf{x}_i(r_1)$$

```
Cylinder
  orientation
    1 2 0
  bounds on the axial variable
    -1. 1.
  bounds on the radial variable
    .5 .75
  boundary conditions
    -1 -1 0 0 3 0
  mappingName
    main-cylinder
  lines
    31 21 6
  exit
Cylinder
  mappingName
    top-cylinder
  orientation
    2 0 1
  bounds on the axial variable
    .25 1.
  bounds on the radial variable
    .3 .4
  boundary conditions
    -1 -1 0 0 3 4
  lines
    25 15 5
  exit
join
 choose curves
   top-cylinder
   main-cylinder (side=0,axis=2)
   compute join
 lines
  25 11 6   31 15 7
 boundary conditions
   -1 -1 1 1 1 0
```



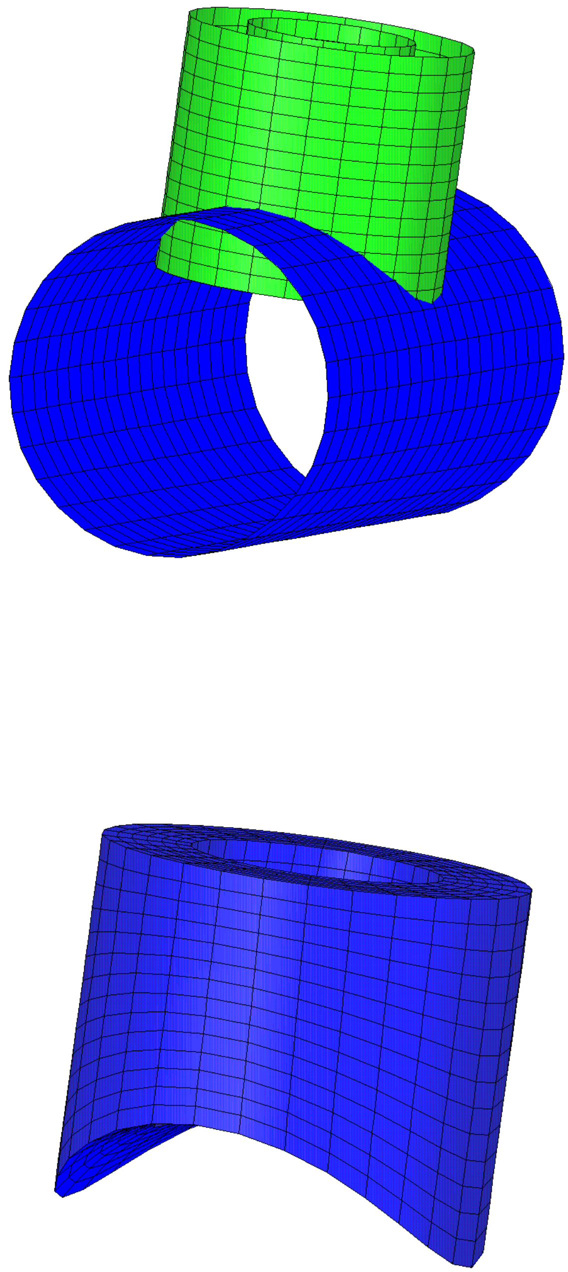
A new surface mapping is generated (the upper 'cylinder' in the bottom figure) that lies on the vertical cylinder and exactly matches to the horizontal cylinder.

17.3 Intersecting a volume intersector mapping with a surface intersectee mapping.

Suppose now that intersector Mapping defines a volume mapping, $R^3 \rightarrow R^3$. The JoinMapping can be used to build a new volume Mapping that will match exactly to the intersectee surface.

In this case we assume that two sides of the intersector mapping intersect the intersectee mapping, say, $\mathbf{C}_i(r_1, r_2, 0)$ and $\mathbf{C}_i(r_1, r_2, 1)$. We proceed as before to generate a `JoinMapping` for each of these intersecting surfaces, $\mathbf{J}_m(\mathbf{r})$, $m = 1, 2$. We also generate a third...

```
Cylinder
  orientation
    1 2 0
  bounds on the axial variable
    -1. 1.
  bounds on the radial variable
    .5 .75
  boundary conditions
    -1 -1 0 0 3 0
  mappingName
    main-cylinder
  lines
    31 21 6
  exit
Cylinder
  mappingName
    top-cylinder
  orientation
    2 0 1
  bounds on the axial variable
    .25 1.
  bounds on the radial variable
    .3 .4
  boundary conditions
    -1 -1 0 0 3 4
  lines
    25 15 5
  exit
join
 choose curves
   top-cylinder
   main-cylinder (side=0,axis=2)
   compute join
 lines
  25 11 6   31 15 7
 boundary conditions
   -1 -1 1 1 1 0
```



A new volume mapping is generated that lies on the vertical cylinder and exactly matches to the horizontal cylinder.

17.4 setEndOfJoin

int
setEndOfJoin(const real & endOfJoin_)

Description: Specify the r value for the end of the join opposite the curve of intersection.

endOfJoin_ (input) : a value in [0,1].

17.5 map

void
map(const realArray & r, realArray & x, realArray & xr, MappingParameters & params)

Purpose: Evaluate the TFI and/or derivatives.

17.6 update

int
update(MappingInformation & mapInfo)

Purpose: Interactively create and/or change the Join mapping.

mapInfo (input): Holds a graphics interface to use.

17.7 Class LineMapping

This mapping a line in one, two or three dimensions.

$$
\begin{aligned}
\mathbf{x}(r) &= x_a + r(x_b - x_a) \\
\mathbf{x}(r) &= (x_a, y_a) + r(x_b - x_a, y_b - y_a) \\
\mathbf{x}(r) &= (x_a, y_a, z_a) + r(x_b - x_a, y_b - y_a, z_b - z_a)
\end{aligned}
$$

17.8 Constructor

LineMapping(const real xa_,
const real xb_,
const int numberOfGridPoints)

Description: Build a mapping for a line in 1D.

xa_, xb_ (input) : End points of the interval.

17.9 Constructor

LineMapping(const real xa_,const real ya_,
const real xb_,const real yb_,
const int numberOfGridPoints)

Description: Build a mapping for a line in 2D.

xa_, ya_, xb_, yb_ (input) : End points of the line.

17.10 Constructor

LineMapping(const real xa_,const real ya_,const real za_,
const real xb_,const real yb_,const real zb_,
const int numberOfGridPoints)

Description: Build a mapping for a line in 3D.

xa_, ya_,za_, xb_, yb_,zb_ (input) : End points of the line.

17.11 getPoints

int
getPoints(real & xa_, real & xb_) const

Description: Get the end points of the line.

xa_, xb_ (output) : End points of the line.

17.12 getPoints

int
getPoints(real & xa_, real & ya_,
real & xb_, real & yb_) const

Description: Get the end points of the line.

xa_, ya_, xb_, yb_ (output) : End points of the line.

17.13 getPoints

int
getPoints(real & xa_, real & ya_, real & za_,
real & xb_, real & yb_, real & zb_) const

Description: Get the end points of the line.

xa_, ya_,za_, xb_, yb_,zb_ (output) : End points of the line.

17.14 setPoints

int
setPoints(const real & xa_, const real & xb_)

Description: Specify the end points for a line in 1D.

xa_, xb_ (input) : End points of the interval.

17.15 setPoints

int
setPoints(const real & xa_, const real & ya_,
const real & xb_, const real & yb_)

Description: Specify the end points for a line in 2D.

xa_, ya_, xb_, yb_ (input) : End points of the line.

17.16 setPoints

int
setPoints(const real & xa_, const real & ya_, const real & za_,
const real & xb_, const real & yb_, const real & zb_)

Description: Specify the end points for a line in 3D.

xa_, ya_,za_, xb_, yb_,zb_ (input) : End points of the line.

18 MatrixMapping: define a mapping from scalings, shifts and rotations

This mapping can be used for rotations, scalings and shifts or any transformation that can be represented as a matrix times a vector. The mapping is defined by a 4×4 matrix that maps from r to x by the relation

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \end{bmatrix} = \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ 1 \end{bmatrix}$$

Each time the functions `rotate`, `scale` and `shift` are called the current `matrix` is updated and thus the transformations are cummulative.

Here is an example of the use of the `MatrixMapping` class.

```
#include "Overture.h"

void main()
{
  realArray r(1,3);
  realArray x(1,3);
  realArray xr(1,3,3);


  MatrixMapping rotScaleShift  ;     // Define a matrix mapping

  rotScaleShift.rotate( axis3, Pi/2. );  // rotate about the x_3 axis
  rotScaleShift.scale( 2.,1.,1. );       // scale by 2 in x_1-direction
  rotScaleShift.shift( 0.,1.,0. );       // shift by 1 in x_2 direction

  r=.5;

  rotScaleShift.map( r,x,xr );

}
```

18.1 Constructor

**MatrixMapping(int domainDimension_ = 3,
int rangeDimension_ = 3)**

Purpose: Build a matrix mapping. This is normally used with the `MatrixTransform` to rotate, scale, or translate an existing mapping.

domainDimension_, rangeDimension_ (input) : domain and range dimension.

18.2 rotate

void
rotate(const int axis, const real theta)

Purpose: Perform a rotation about a given axis. This rotation is applied after any existing transformations. Use the reset function first if you want to remove any existing transformations.

axis (input) : axis to rotate about (0,1,2)

theta (input) : angle in radians to rotate by.

18.3 rotate

void
rotate(const RealArray & rotate)

Purpose: Perform a ”rotation” using a 3×3 matrix. This does not really have to be a rotation. This transformation replaces any existing transformation.

rotate (input): The upper 3×3 portion of the 4×4 transformation matrix will be replaced by the matrix `rotate(0:2,0:2)`.

18.4 scale

void
scale(const real scalex =1.,
const real scaley =1.,
const real scalez =1.)

Purpose: Perform a scaling

scalex, scaley, scalez (input): Scale factors along each axis.

18.5 shift

void
shift(const real shiftx =0.,
const real shifty =0.,
const real shiftz =0.)

Purpose: Perform a shift.

shitx, shity, shitz (input): shifts along each axis.

18.6 reset

void
reset()

Purpose: reset the matrix to the identity.

18.7 matrixMatrixProduct

void
matrixMatrixProduct(RealArray & m1, const RealArray & m2, const RealArray & m3)

Purpose: Multiply two 4x4 matrices together. This is a utility routine (a static member function that can be called without a MatrixMapping object using MatrixMapping::matrixMatrixProduct(...)).

```
m1 <- m2*m3
```

18.8 matrixVectorProduct

void
matrixVectorProduct(RealArray & v1, const RealArray & m2, const RealArray & v3)

Purpose: Multiply a 4x4 matrix times a vector. This is a utility routine (a static member function).

```
v1 <- m2*v3
```

18.9 matrixInversion

int
matrixInversion(RealArray & m1Inverse, const RealArray & m1)

Purpose: Invert a 4x4 matrix. This is a utility routine (a static member function). This only works for matrices used in transforming 3D vectors which look like:

```
[ a00 a01 a02 a03 ]
[ a10 a11 a12 a13 ]
[ a20 a21 a22 a23 ]
[  0   0   0   1  ]
```

Return value: 0=success, 1=matrix is not invertible

19 MatrixTransform: rotate, scale or shift an existing mapping

19.1 Description

The `MatrixTransform` mapping can be used to rotate, scale and shift another mapping. It does this by composing the given mapping with a `MatrixMapping`, section (18).

19.2 Constructor

MatrixTransform()

Purpose: Build a mapping for matrix transform.

19.3 Constructor(Mapping&)

MatrixTransform(Mapping & map)

Purpose: Build a Mapping for matrix transformation of another Mapping.

19.4 reset

void
reset()

Purpose: Reset the transformation to the identity.

19.5 rotate

void
rotate(const int axis, const real theta)

Purpose: Perform a rotation about a given axis.

axis (input) : axis to rotate about (0,1,2)

theta (input) : angle in radians to rotate by.

19.6 rotate

void
rotate(const RealArray & rotate)

Purpose: Perform a rotation using a 3×3 "rotation" matrix. This does not really have to be a rotation.

rotate (input): The upper 3×3 portion of the 4×4 transformation matrix will be replaced by the matrix `rotate(0:2,0:2)`.

19.7 scale

void
scale(const real scalex =1.,
const real scaley =1.,
const real scalez =1.)

Purpose: Perform a scaling

scalex, scaley, scalez (input): Scale factors along each axis.

19.8 shift

void
shift(const real shiftx =0.,
const real shifty =0.,
const real shiftz =0.)

Purpose: Perform a shift.

shitx, shity, shitz (input): shifts along each axis.

20 NormalMapping: define a new mapping by extending normals

20.1 Description

The `NormalMapping` extends normals from an existing curve or surface to generate a new mapping.

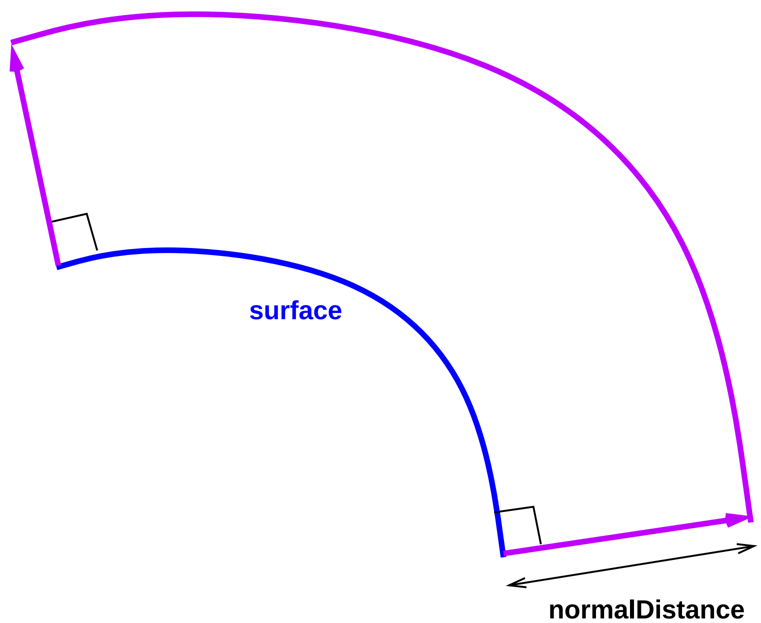


Figure 11: The NormalMapping defines a new mapping by extending normals from a given curve or surface

20.2 Member Functions

ormalMappingInclude.tex

21 NurbsMapping: define a new mapping as a NURBS.

******* This class is still under development ********

The `NurbsMapping` class defines mapings in terms of a non-uniform rational b-spline, NURBS. The implementation here is based on the reference, *The NURBS Book* Les Piegl and Wayne Tiller, Springer, 1997.

The n-th degree Berstein polynomial is

$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}$$

and the n-th degree Bezier curve

$$\mathbf{C}(u) = \sum_{i=0}^{n} B_{i,n}(u)\mathbf{P}_i, \qquad 0 \le u \le 1$$

with control points $\mathbf{P}_i$.

The n-th degree rational Bezier curve is

$$\begin{aligned}\mathbf{C}(u) &= \frac{\sum_{i=0}^{n} B_{i,n}(u) w_i \mathbf{P}_i}{\sum_{i=0}^{n} B_{i,n}(u) w_i}, \qquad 0 \le u \le 1 \\ &= \sum_{i=0}^{n} R_{i,n}(u)\mathbf{P}_i\end{aligned}$$

with weights w_i.

Written using homogeneous coordinates

$$\mathbf{C}^w(u) = \sum_{i=0}^{n} B_{i,n}(u)\mathbf{P}_i^w$$

where $\mathbf{P}_i^w = (w_i\mathbf{P}_i, w_i)$.

B-spline basis functions are defined as

$$\begin{aligned}N_{i,0}(u) &= \begin{cases} 1 & u_i \le u < u_{i+1} \\ 0 & \text{otherwise} \end{cases} \\ N_{i,p}(u) &= \frac{u-u_i}{u_{i+p}-u_i} N_{i,p-1}(u) + \frac{u_{i+p+1}-u}{u_{i+p+1}-u_{i+1}} N_{i+1,p-1}(u)\end{aligned}$$

where $\mathbf{U} = \{u_0, \ldots, u_m\}$ are the knots, $u_i \le u_{i+1}$.

We only use nonperiodic (clamped or open) knot vectors,

$$\mathbf{U} = \{a, \ldots, a, u_{p+1}, \ldots, u_{m-p-1}, b, \ldots, b\}$$

with the end knots repeated $p+1$ times.

NonUniform Rational B-Spline (NURBS). p-th degree NURBS curve

$$\begin{aligned}\mathbf{C}(u) &= \frac{\sum_{i=0}^{n} N_{i,p}(u) w_i \mathbf{P}_i}{\sum_{i=0}^{n} N_{i,p}(u) w_i}, \qquad a \le u \le b \\ &= \sum_{i=0}^{n} R_{i,p}(u)\mathbf{P}_i\end{aligned}$$

Written using homogeneous coordinates

$$\mathbf{C}^w(u) = \sum_{i=0}^{p} N_{i,p}(u)\mathbf{P}_i^w$$

21.1 Constructor

NurbsMapping()

Purpose: Default Constructor, make a null NURBS.

Remarks: The implementation here is based on the reference, *The NURBS Book* by Les Piegl and Wayne Tiller, Springer, 1997. The notation here is:

- degree = p (variables p1,p2 for one and 2D)
- number of control points is n+1 (variables n1,n2)
- number of knots is m+1 (m=n+p+1) (variables m1,m2)
- cPoint(0:n,0:r) : holds the control points and weights. r=rangeDimension.
- uKnot(0:m) : holds knots along axis1. These are normally scaled to [0,1] (see notes below).
- vKnot(0:m) : holds knots along axis2 (if domainDimension==2)
- note : Knots are scaled to [0,1]

NOTES: for those wanting to make changes to this class

uMin,uMax,vMin,vMax : A typical NURBS will have knots that span an arbitrary interval. For example the knots may go from $[.5, 1.25]$. This mapping however, is parameterized on [0,1]. To fix this we first save the actual min and max values for uKnot in [uMin,uMax] and similarly for [vMin,vMax]. We then rescale uKnot and vKnot to lie on the interval [0,1]. Note that the `reparameterize` function may subsequently rescale the knots to a larger interval in which case the NURBS will only represent a part of the initial surface. If we do this then we also rescale uMin,uMax,vMin,vMax. The `parametricCurve` function is used to indicate that this NURBS is actually a parametric curve on another NURBS, nurbs2. By default the values of uMin,uMax,vMin,vMax from nurbs2 are used to scale this NURBS in order to make it compatible with the rescaled nurbs2.

21.2 Constructor

NurbsMapping(const int & domainDimension_ , const int & rangeDimension_)

Purpose: Constructor, make a default NURBS of the give domain dimension (1,2)

21.3 intersect3DLines

int
intersect3DLines(realArray & pt0, realArray & t0,
realArray & pt1, realArray & t1,
real & alpha0, real & alpha1,
realArray & pt2)

Description: Intersect two lines in 3D: x0(s) = pt0 + s * t0 x1(t) = pt1 + t * t1

alpha0,alpha1 : values of s and t at the intersection.

pt2 : point of intrsection, x0(alpha0)=pt2=x1(alpha1)

Return values: 1 if the line are parallel, 0 otherwise.

21.4 buildCurveOnSurface

int
buildCurveOnSurface(NurbsMapping & curve,
real r0,
real r1 =1.)

Description: Build a new Nurbs curve that matches a coordinate line on the surface.

curve (output) : on output a curve that matches a coordinate line on the surface.

r0,r1 (input) : if r1==-1 make a curve $\mathbf{c}(r) = \mathbf{s}(r0, r)$ where $\mathbf{s}(r_0, r_1)$ is the NURBS surface defined by this mapping. If r0==-1 make the curve $\mathbf{c}(r) = \mathbf{s}(r, r1)$ the arc, measured starting from x.

21.5 circle

int
circle(realArray & o,
realArray & x,
realArray & y,
real r,
real startAngle =0.,
real endAngle =1.)

Description: Build a circular arc. Reference the NURBS book Algorithm A7.1

o (input): center of the circle.

x,y (input): orthogonal unit vectors in the plane of the circle.

startAngle,endAngle : normalized angles [0,1] for the start and end of the arc, measured starting from x.

21.6 getKnots

const realArray &
getKnots(int direction =0) const

Purpose: get uKnot or vKnot, the knots in the first or second direction.

direction: 0=return uKnot, 1= return vKnot.

21.7 getControlPoints

const realArray &
getControlPoints() const

Purpose: Return the control points, scaled by the weight.

21.8 insertKnot

int
insertKnot(const real & uBar,
const int & numberOfTimesToInsert_ =1)

Purpose: Insert a knot

uBar (input): Insert this knot value.

numberOfTimesToInsert_ (input): insert the knot this many times. The multiplicity of the knot will not be allowed to exceed p1.

21.9 insertKnot

int
normalizeKnots()

Access: Protected routine.

Purpose: Normalize the knots, uKnot (and vKnot if domainDimension==2) to lie from 0 to 1. This routine will NOT change the values of uMin,uMax, vMin,vMax since these values indicate the original bounds on uKnot and vKnot.

21.10 readFromIgesFile

int
readFromIgesFile(IgesReader & iges, const int & item, bool normKnots /*=true*/)

Purpose: Read a NURBS from an IGES file. An IGES file is a data file containing geometrical objects, usually generated by a CAD program.

iges (input) : Use this object to read the IGES file.

item (input) : read this item from the IGES file.

21.11 parametricCurve

int
parametricCurve(const NurbsMapping & nurbs,
const bool & scaleParameterSpace = TRUE)

Purpose: Indicate that this nurb is actually a parametric curve on another nurb surface.

nurbs (input) : Here is the NURBS surface for which this NURBS is a parametric surface.

scaleParameterSpace (input) : if TRUE, scale the range space of this nurb to be on the unit interval. This is usually required since the NurbsMapping scales the knots to lie on [0,1] (normally) and so we then need to scale this Mapping to be consistent.

21.12 shift

int
shift(const real & shiftx =0.,
const real & shifty =0.,
const real & shiftz /* =0.*/)

Purpose: Shift the NURBS in space.

21.13 scale

int
scale(const real & scalex =0.,
const real & scaley =0.,
const real & scalez /* =0.*/)

Purpose: Scale the NURBS in space.

21.14 rotate

int
rotate(const int & axis, const real & theta)

Purpose: Perform a rotation about a given axis. This rotation is applied after any existing transformations. Use the reset function first if you want to remove any existing transformations.

axis (input) : axis to rotate about (0,1,2)

theta (input) : angle in radians to rotate by.

21.15 rotate

int
matrixTransform(const RealArray & m)

Purpose: Perform a general matrix transform using a 2x2 or 3x3 matrix. Convert the NURBS to 2D or 3D if the transformation so specifies – i.e. if you transform a NURBS with rangeDimension==2 with a 3x3 matrix then the result will be a NURBS with rangeDimension==3.

m (input) : m(0:2,0:2) matrix to transform with

21.16 specify knots and control points

int
specify(const int & m,
const int & n,
const int & p,
const realArray & knot,
const realArray & controlPoint,
const int & rangeDimension_ =3,
bool normalizeTheKnots /* =true*/)

Purpose: Specify a **curve** in 2D or 3D using knots and control points

m (input) : The number of knots is m+1

n (input) : the number of control points is n+1

p (input) : order of the B-spline

controlPoint(0: n,0:rangeDimension) (input) : control points and weights

normalizeTheKnots (input) : by default, normalize the knots to [0,1]. Set to false if you do not want the knots normalized.

21.17 specify knots and control points

int
specify(const int & n1_,
const int & n2_,
const int & p1_,
const int & p2_,
const realArray & uKnot_,
const realArray & vKnot_,
const realArray & controlPoint,
const int & rangeDimension_ =3,
bool normalizeTheKnots /* =true*/)

Purpose: Specify a NURBS with domainDimension==2 using knots and control points

n1_,n2_ (input) : the number of control points is n1+1 by n2+1

p1_,p2_ (input) : order of the B-spline in each direction.

uKnot_,vKnot_ (input) : knots.

controlPoint(0: n1,0:n2,0:rangeDimenion) (input) : control points and weights

normalizeTheKnots (input) : by default, normailize the knots to [0,1]. Set to false if you do not want the knots normalized.

21.17.1 setDomainInterval

int
setDomainInterval(const real & r1Start =0.,
const real & r1End =1.,
const real & r2Start =0.,
const real & r2End =1.,
const real & r3Start =0.,
const real & r3End =1.)

Description: Restrict the domain of the nurbs. By default the nurbs is parameterized on the interval [0,1] (1D) or [0,1]x[0,1] in 2D etc. You may choose a sub-section of the nurbs by choosing a new interval [rStart,rEnd]. For periodic nurbss the interval may lie in [-1,2] so the sub-section can cross the branch cut. You may even choose rEnd¡rStart to reverse the order of the parameterization.

rStart1,rEnd1,rStart2,rEnd2,rStart3,rEnd3 (input) : define the new interval.

21.18 initialize()

void
initialize( )

Purpose: Initialize the NURBS. This is a protected routine. Determine if the weights are constant so that we can use more efficient routines. Set bounds for the Mapping.

NOTES: Normally we multiply the control points by the weights. BUT, if the weights are constant we divide everything by this constant value so we can avoid dividing by the weight term when we evaluate. When the weights are constant `nonUniformWeights==false`;

21.19 setBounds

void
setBounds()

Purpose: protected routine. Set the approximate bounds on the mapping, used by plotting routines etc. Use the control points as an approximation *** note only apply this to the normalized control-points ***

21.20 removeKnot

int
removeKnot(const int & index,
const int & numberOfTimesToRemove,
int & numberRemoved, const real & tol)

Purpose: Remove a knot (if possible) so that the Nurbs remains unchanged

index (input) : try to remove the knot at this index

numberOfTimesToRemove (input) : the number of times to try and remove the knot.

numberRemoved (output): the actual number of times the knot was removed

21.21 getParameterBounds

int
getParameterBounds(int axis, real & rStart_, real & rEnd_) const

Purpose: Return current values for the parameter bounds.

axis (input) : return bounds for this axis.

rStart_, rEnd_: bounds.

21.22 reparameterize

int
reparameterize(const real & uMin_,
const real & uMax_,
const real & vMin_ =0.,
const real & vMax_ =1.)

Purpose: Reparameterize the nurb to only use a sub-rectangle of the parameter space. This function can also be used to reverse the direction of the parameterization by choosing uMin $>$ uMax and/or vMin $>$ vMax.

uMin,uMax (input): subrange of u values to use, normally $0 \leq$ uMin $\neq$ uMax ≤ 1

vMin,vMax (input): subrange of v values to use, normally $0 \leq$ vMin $\neq$ vMax ≤ 1 (for domainDimension==2)

Notes: this routine just scales the knots to be on a larger interval than [0,1]. Thus when the Mapping is evaluated on [0,1] the result will only be a portion of the original surface.

Return values: 0 : success, 1 : failure

21.23 transformKnots

int
transformKnots(const real & uScale,
const real & uShift,
const real & vScale =1.,
const real & vShift =0.)

Purpose: Apply a scaling and shift to the to the knots: uScale*uKnots+uShift. The scale factors should be positive.

uScale,uShift (input): scaling and shift for the knots in the u direction.

vScale,vShift (input): scaling and shift for the knots in the v direction. (for domainDimension==2)

21.24 elevateDegree

int
elevateDegree(const int increment)

Purpose: Elevate the degree of the nurbs.

increment (input): increase the degree of the nurb by this amount ¿=0

Return values: 0 : success, 1 : failure

21.25 merge

int
merge(NurbsMapping & nurbs, bool keepFailed = true, real eps /*=-1*/, bool attemptPeriodic /*=true*/)

Purpose: Try to merge "this" nurbs with the input nurbs. This routine will merge the two NURBS's into one if the endpoint of one matches the end point of the second.

nurbs (input): nurbs to merge with

Return values: 0 : success, 1 : failure

21.26 forcedMerge

int
forcedMerge(NurbsMapping & nurbs)

Purpose: Force a merge of "this" nurbs with the input nurbs. This routine will merge the two NURBS's into one if the endpoint of one matches the end point of the second. If the endpoints do not match, a straight line section is added between the closest end points.

nurbs (input): nurbs to merge with

Return values: 0 : success, 1 : failure

21.27 forcedPeriodic

int
forcePeriodic()

Purpose: force this mapping to be periodic by making the last control points the same as the first (if the knots are "clamped", eg the knots are 0 0 0 0 ... 1 1 1 1)

Return values: 0 : success, 1 : failure

21.28 split

int
split(real uSplit, NurbsMapping &c1, NurbsMapping&c2)

Description: Split a nurb curve into two pieces.

uSplit (input) : parameter value to split the curve at

c1 (output) : curve on the "left", parameter bounds [0,uSplit]

c2 (output) : curve on the "right", parameter bounds [uSplit,1]

Returns : 0 on success, 1 on failure (uSplit¡0 or uSplit¿1)

21.29 moveEndpoint

int
moveEndpoint(int end, const realArray &endPoint, real tol /*=-1*/)

Description: Move either the beginning or the end of the curve to endPoint.

21.30 numberOfSubCurves

int
numberOfSubCurves() const

Description: If the Nurb is formed by merging a sequence of Nurbs then function will return that number. By default the numberOfSubCurves would be 1 if no Nurbs were merged.

21.31 numberOfSubCurvesInList

int
numberOfSubCurvesInList() const

Description: Return the number of subcurves used to build the Nurb plus the number of hidden curves By default the numberOfSubCurvesInList would be 1 if no Nurbs were merged.

21.32 subCurve

NurbsMapping&
subCurve(int subCurveNumber)

Description: If the Nurb is formed by merging a sequence of Nurbs then function will return that Nurbs. If the numberOfSubCurves is 1 then the current (full) Nurbs is returned.

21.33 subCurveFromList

NurbsMapping &
subCurveFromList(int subCurveNumber)

Description: Return a nurb curve directly from the list of subcurves. This can be a curve used to generate the nurb itself or one of the ”hidden” curves. If the numberOfSubCurves is 1 then the current (full) Nurbs is returned.

21.34 interpolate

void
interpolate(const realArray & x,
const int & option = 0,
realArray & parameterization =Overture::nullRealDistributedArray(),
int degree = 3)

Purpose: Define a new NURBS curve that interpolates the points x(0:n1,0:r-1) OR define a new NURBS surface that interpolates the points x(0:n1,0:n2,0:r-1) (NEW feature). By default the NURBS curve will be parameterized by a the chord length.

option (input) : if(option==0 then use the array parameterization. if option==1 then return the parameterization used in the array parameterization.

parameterization_(0: n1) (input) : optionally specify the parameterization. These values should start from 0, end at 1 and be increasing. If this argument is not given then the parameterization will be based on chord length. If option==1 then the actual parameterization used will be returned in this array.

degree (input) : degree of approximation. Normally a value such as 1,2,3.

21.35 map

// void

//===

/ /Purpose: Evaluate the nurbs and/or derivatives.

21.36 mapVector

// void

//===

/ /Purpose: Evaluate the nurbs and/or derivatives. This routine is a // version of the `map` function that is optimized for vectors of points.

21.37 put(fileName)

int
put(const aString & fileName, const FileFormat & fileFormat = xxww)

Description: put NURBS data into an ascii readable file.

fileName (input) : name of the file.

fileFormat (input) : specify the file format. (see the comments with the get(const aString&,...) function).

21.38 put(FILE*)

int
put(FILE *file, const FileFormat & fileFormat = xxww)

Description: Save the NURBS data to an ascii readable file.

fileFormat (input) : specify the file format. (see the comments with the get(const aString&,...) function).

21.39 get(fileName)

int
get(const aString & fileName, const FileFormat & fileFormat = xxww)

Description: read NURBS data from an ascii readable file.

fileName (input) : get from this file.

fileFormat (input) : specify the file format.

Here is the file format for `fileFormat=xxww` for a surface in 3D

```
domainDimension rangeDimension p1 n1 p2 n2
uKnot(0) uKnot(1) ... uKnot(m1)  --- on possibly multiple lines, at most 10 values per li
vKnot(0) vKnot(1) ... vKnot(m2)
x0 x1 x2 ...            --- x coords of control pts. on multiple lines, at most 10 per li
y0 y1 y2 ...            --- y coords of control pts.
z0 z1 z2 ...            --- z coords of control pts.
w0 w1 w2 ...            --- weights of control pts.
```

If the domainDimension==1 then leave off p2 and n2. If the rangeDimension is 2 then leave off the z values. Here m1=n1+p1+1 and m2=n2+p2+1.

Here is the file format for `fileFormat=xwxw` for a surface in 3D

```
domainDimension rangeDimension p1 n1 p2 n2
uKnot(0) uKnot(1) ... uKnot(m1)  --- on possibly multiple lines, at most 10 values per li
vKnot(0) vKnot(1) ... vKnot(m2)
x0 y0 z0 w0                   --- control point 0
x1 y1 z1 w1                   --- control point 1
x1 y1 z1 w1                   --- control point 2
...
```

If the domainDimension==1 then leave off p2 and n2. If the rangeDimension is 2 then leave off the z values.

21.40 put(FILE *)

int
get(FILE *file, const FileFormat & fileFormat = xxww)

Description: read NURBS data from an ascii readable file.

file (input) : get from this file.

fileFormat (input) : specify the file format. (see the comments with the get(const aString&,...) function).

21.41 getOrder

int
getOrder(int axis =0) const

Purpose: Return the order, p.

21.42 getNumberOfKnots

int
getNumberOfKnots(int axis =0) const

Purpose: Return the number of knots, m+1.

21.43 getNumberOfControlPoints

int
getNumberOfControlPoints(int axis =0) const

Purpose: Return the number of control points, n+1.

21.44 buildSubCurves

int
buildSubCurves(real angle =60.)

Purpose: Split a NURBS curve at corners into sub-curves. Currently this only applies if the order of the NURBS is 1 (piece-wise linear).

angle (input) : divide the curve at points where the tangent changes by more than this angle (degrees)

21.45 truncateToDomainBounds

int
truncateToDomainBounds()

Purpose: clip the knots and control polygon to the bounds set in rstart and rend

21.46 toggleSubCurveVisibility

int
toggleSubCurveVisibility(int sc)

Description: Toggle a subcurve's "visibility", a visible subcurve is accessible through NurbsMapping::subCurve(..) method an invisible subcurve is only accessible through NurbsMapping::subCurveFromList()

sc (input) : the subcurve to toggle Returns : the new subcurve number NOTES : this will reorder the subcurves in the subCurves array

21.47 isSubCurveHidden

bool
isSubCurveHidden(int sc)

Description: find out if a subcurve is hidden or not, returns true if hidden, false if visible

sc (input) : the subcurve to querry

21.48 isSubCurveOriginal

bool
isSubCurveOriginal(int sc)

Description: find out if a subcurve is marked as "original"

sc (input) : the subcurve to querry

21.49 toggleSubCurveOriginal

void
toggleSubCurveOriginal(int sc)

Description: toggle the ”original” status on a subcurve, ”original” is just a marker used to distingish the original subcurves used to build this nurb from subsequent modifications.

sc (input) : the subcurve to alter

21.50 addSubCurve

int
addSubCurve(NurbsMapping &nurbs)

Description: Add a subcurve to this mapping. Note that the nurb is copied and is set to visible. The ”original” marker is set to false;

Returns : the index of the new curve in the list of visible curves

21.51 deleteSubCurve

int
deleteSubCurve(int sc)

Description: Delete a subcurve from the list of curves. Note this shifts the subcurve list making previous indices invalid

sc (input): the curve to delete

Returns : 0 on success

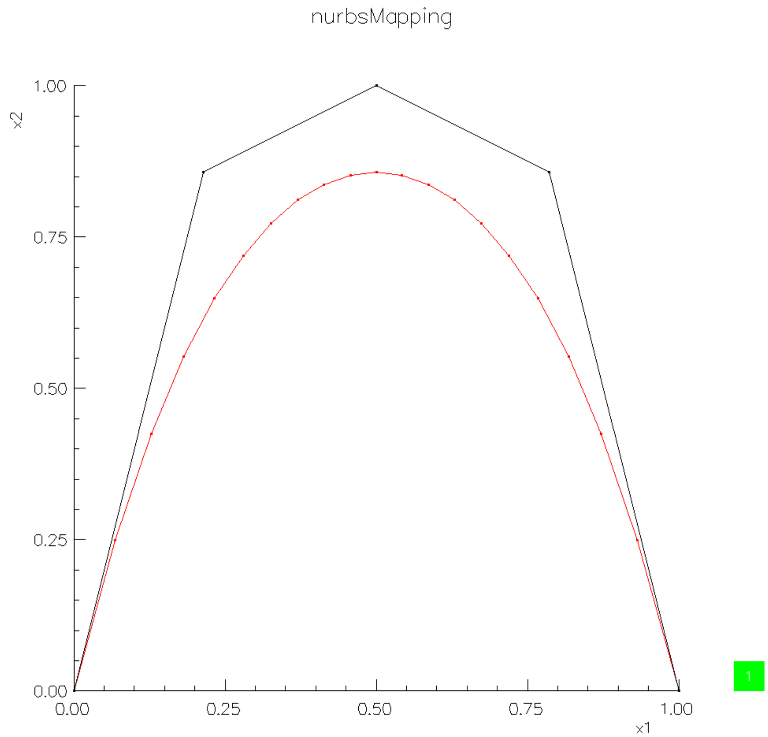
21.52 update

int
update(MappingInformation & mapInfo)

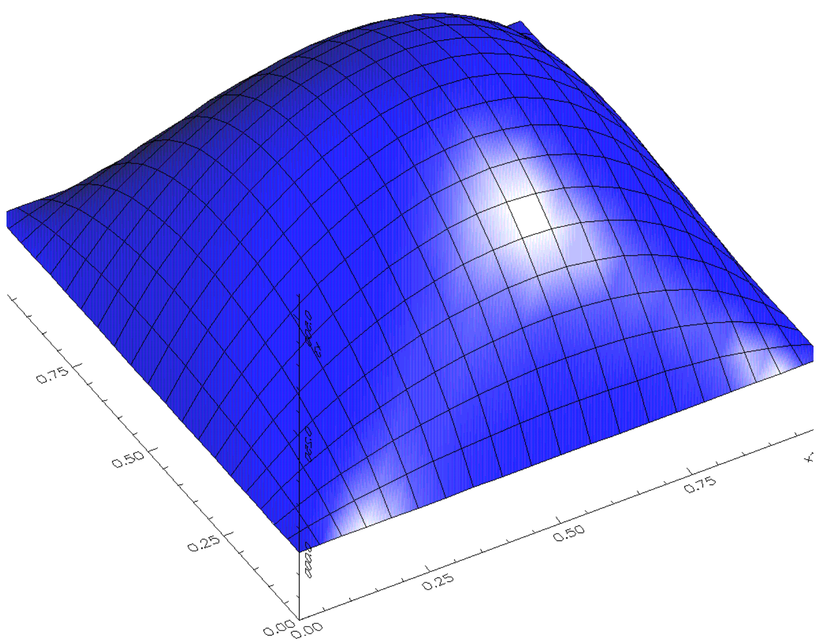
Purpose: Interactively create and/or change the nurbs mapping.

mapInfo (input): Holds a graphics interface to use.

21.53 Examples



A 2D NURBS curve defined by specifying control points.



A 3D NURBS surface defined by specifying control points.

22 OffsetShell: Define mappings to build a grid around a shell or plate.

The OffsetShell class starts with a 3D surface defining a thin shell or plate (this is called the reference surface). An offset surface will be built by translating the reference surface a small amount in a user specified direction. An edge surface will then be constructed that joins the reference and offset surfaces with a rounded edge that overlaps both surfaces.

Volume grids can be built for the reference, offset and edge surfaces.

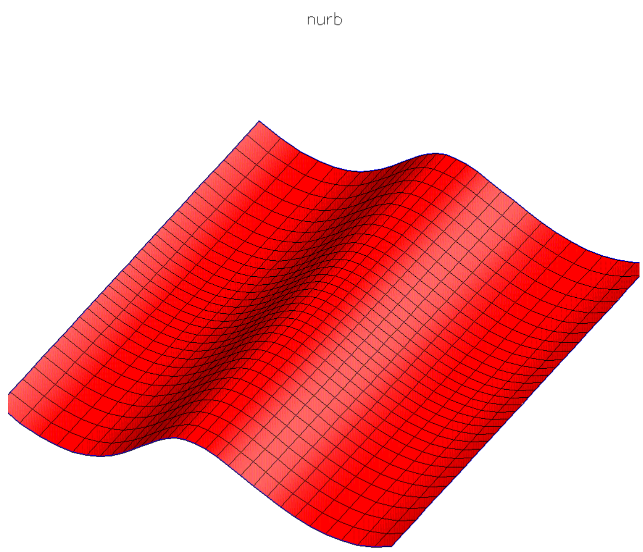


Figure 12: The original reference surface for a flying carpet given to OffsetShell.

22.1 Defining the edge surface : an overlapping round

The first step in defining the edge surface is to define a smooth curve on the reference surface that smoothly follows the boundary of the reference surface but is offset a small amount inside the boundary. To define this curve we first construct a smooth curve, $\mathbf{c}_0(t) = (r_0, r_1)(t)$, near the boundary of the unit square:

$$
\mathbf{c}_0(t) = \begin{cases} (1-\Delta_0, .5+\xi) & 0 \le \xi \le t_0 \\ (1-\Delta_1-\xi, 1-\Delta_0) & t_1 \le \xi \le t_2 \\ (\Delta_0, 1-\Delta_1-\xi) & t_3 \le \xi \le t_4 \\ (\Delta_1+\xi, \Delta_0) & t_5 \le \xi \le t_6 \\ (1-\Delta_0, \Delta_1+\xi) & t_7 \le \xi \le t_8 \end{cases}
$$

$$
t_0 = .5 - \Delta_1
$$

The curve on the reference surface is defined as $\mathbf{c}(t) = \mathbf{R}(\mathbf{c}_0(t))$ where $\mathbf{x}(\mathbf{r}) = \mathbf{R}(\mathbf{r})$ defines the reference surface.

Given the edge curve $\mathbf{c}(t)$ we can define the tangent vector $\mathbf{t}(t)$ to the curve as well as the vector normal to the reference surface, $\mathbf{n}(t)$.

$$
\mathbf{t}(t) = \dot{\mathbf{c}}/\|\dot{\mathbf{c}}\|
$$

$$
\mathbf{n}(t) = \frac{\partial \mathbf{R}}{\partial r_0}(\mathbf{c}_0(t)) \times \frac{\partial \mathbf{R}}{\partial r_1}(\mathbf{c}_0(t))
$$

Given $\mathbf{t}(t)$ and $\mathbf{n}(t)$ we define the direction vector, $\mathbf{d}(t)$, at each point on the edge curve to be orthogonal to these two vectors and point towards the boundary of the reference surface,

$$
\mathbf{d}(t) = \mathbf{t} \times \mathbf{n}/\|\mathbf{t} \times \mathbf{n}\|
$$

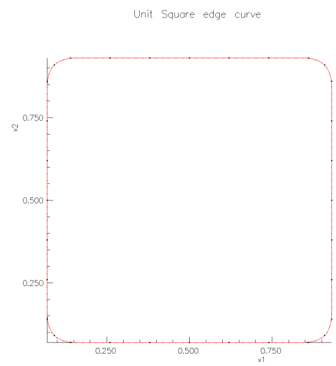


Figure 13: An edge curve $\mathbf{c}_0(t)$ is defined on the unit square.

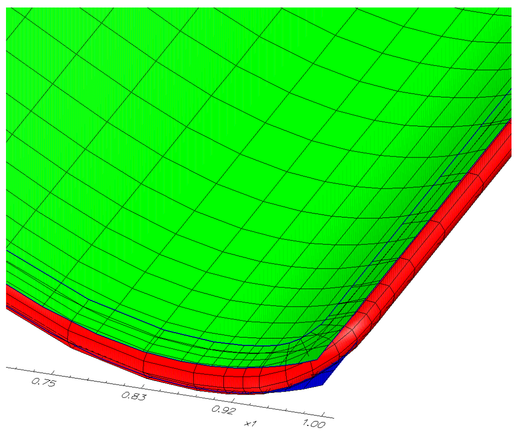


Figure 14: The reference surface, offset surface, and edge surface near a corner.

The edge surface defined as 3 sections, an initial and final portion that lie on the reference surface connected by half a circle:

$$\mathbf{e}(t,s) = \begin{cases} \mathbf{c}(t) + a_0 s \mathbf{d}(t) & 0 \le s \le s_0 \\ \mathbf{e}(t,s_0) + .5(1 - cos(\theta))\mathbf{s} + \sin(\theta) a_1 \mathbf{d}(t) & s_0 < s \le s_1 \\ \mathbf{e}(t,s_1) - a_0(1-s)\mathbf{d}(t) & s_1 < s \le 1 \end{cases}$$
$$\theta = \pi(s - s_0)/(s_1 - s_0)$$

22.2 Member function descriptions

22.3 Constructor

OffsetShell()

Description: Starting from a 3D reference surface build an offset surface and joining edge surface.

22.4 buildOffsetMappings

int
buildOffsetMappings(GenericGraphicsInterface & gi,
GraphicsParameters & parameters, MappingInformation & mapInfo)

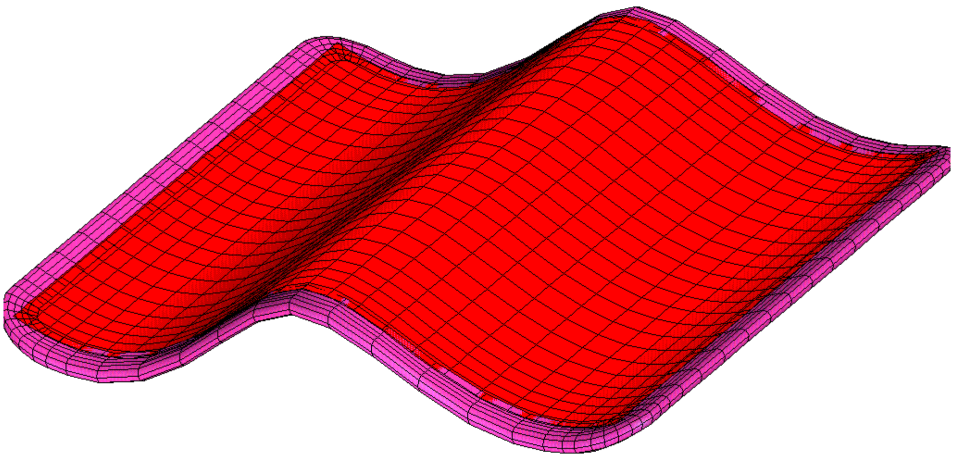


Figure 15: The overlapping grid for the flying carpet in a box.

Description: Given a reference surface, build an offset surface, and then an edge surface to join two. Build volume grids for the reference surface, offset surface and edge surface.

referenceSurface (input) :

offsetSurface (output) :

edgeSurface, edgeVolume (output):

referenceVolume, offsetVolume (output) :

22.5 generateVolumeGrids

int
generateVolumeGrids(GenericGraphicsInterface & gi,
GraphicsParameters & parameters, MappingInformation & mapInfo)

Description: Build the volume grids.

22.6 createOffsetMappings

int
createOffsetMappings(MappingInformation & mapInfo)

Description: Interactively build grids for a thin shell.

23 OrthographicTransform : define an orthographic transform

This mapping is used to create an orthographic patch to remove a spherical-polar singularity or a cylindrical polar singularity (i.e. a singular may occur at the end of a mapping defined in cylindrical coordinates when the cross-sections converge to a point). Normally one would use the `ReparamertizationTransform` mapping to construct the Orthographic patch.

23.1 Description

The orthographic transformation is a mapping from parameter space to parameter space. There are two forms to this mapping, the first can be used to reparameterize a mapping with a spherical polar singularity and the second can be used for a cylindrical mapping with a polar singularity.

23.1.1 Orthographic transform to reparameterize a spherical-polar singularity

This form of the orthographic mapping transforms into spherical polar coordinates

$$(r_1, r_2) \rightarrow (t_1, t_2) = (\frac{\phi}{\pi}, \frac{\theta}{2\pi})$$

and is defined by

$$s_1 = (r_1 - \frac{1}{2})s_a, \quad s_2 = (r_2 - \frac{1}{2})s_b, \quad \sigma^2 = s_1^2 + s_2^2$$

$$\cos\phi = \pm\frac{1-\sigma^2}{1+\sigma^2}, \quad \sin\phi = \frac{2\sigma}{1+\sigma^2}, \quad \cos\theta = \frac{s_1}{\sigma}, \quad \sin\theta = \pm\frac{s_2}{\sigma}.$$

$$(t_1, t_2) = (\frac{\phi}{\pi}, \frac{\theta}{2\pi})$$

The upper sign $(+)$ is used for a reparametrization covering the north pole and the lower sign $(-)$ for the south pole.

The derivatives are returned as

$$\frac{\partial t_1}{\partial r_1} = \frac{s_1}{(1+\sigma^2)\sigma}\frac{\pm 2s_a}{\pi} \tag{11}$$

$$\frac{\partial t_1}{\partial r_2} = \frac{s_2}{(1+\sigma^2)\sigma}\frac{\pm 2s_b}{\pi} \tag{12}$$

$$\sin(\phi)\frac{\partial t_2}{\partial r_1} = -\frac{s_2}{(1+\sigma^2)\sigma}\frac{\pm 2s_a}{\pi} \tag{13}$$

$$\sin(\phi)\frac{\partial t_2}{\partial r_2} = +\frac{s_1}{(1+\sigma^2)\sigma}\frac{\pm 2s_b}{\pi} \tag{14}$$

so that when this mapping is composed with a mapping in spherical-polar form the $\sin(\phi)$ terms will cancel nicely to remove the removable singularity.

The inverse of the mapping $(t_1, t_2) \rightarrow (r_1, r_2)$ is defined by

$$\phi = \pi t_1, \quad \theta = 2\pi t_2, \quad s_1 = \frac{\sin\phi}{1 \pm \cos\phi}\cos\theta, \quad s_2 = \pm\frac{\sin\phi}{1 \pm \cos\phi}\sin\theta.$$

$$r_1 = \frac{s_1}{s_a} + \frac{1}{2}, \quad r_2 = \frac{s_2}{s_b} + \frac{1}{2}$$

The derivatives are returned as

$$\frac{\partial r_1}{\partial t_1} = \frac{\cos(\theta)}{(1 \pm \cos(\phi))}\frac{\pm\pi}{s_a}$$

$$\frac{\partial r_2}{\partial t_1} = \frac{\sin(\theta)}{(1 \pm \cos(\phi))}\frac{\pi}{s_b}$$

$$\frac{1}{\sin(\phi)}\frac{\partial r_1}{\partial t_2} = \frac{\sin(\theta)}{(1 \pm \cos(\phi)}\frac{-2\pi}{s_a}$$

$$\frac{1}{\sin(\phi)}\frac{\partial r_2}{\partial t_2} = \frac{\cos(\theta)}{(1 \pm \cos(\phi)}\frac{\pm 2\pi}{s_b}$$

23.1.2 Orthographic transform to reparameterize a cylindrical polar singularity

This form of the orthographic mapping transforms into cylindrical coordinates

$$(r_1, r_2) \rightarrow (t_1, t_2) = (s, \frac{\theta}{2\pi})$$

and is defined by

$$s_1 = (r_1 - \frac{1}{2})s_a, \quad s_2 = (r_2 - \frac{1}{2})s_b \quad \sigma^2 = s_1^2 + s_2^2$$

$$t_1 = \pm\frac{1}{2}\,\frac{1-\sigma^2}{1+\sigma^2} + \frac{1}{2}, \quad \tan(2\pi t_2) = \pm s_2/s_1, \quad r = \frac{2\sigma}{1+\sigma^2}$$

The derivatives are returned as

$$\begin{aligned} -\frac{1}{r}\frac{\partial t_1}{\partial r_1} &= \frac{s_1}{(1+\sigma^2)\sigma}\pm s_a \\ -\frac{1}{r}\frac{\partial t_1}{\partial r_2} &= \frac{s_2}{(1+\sigma^2)\sigma}\pm s_b \\ r\frac{\partial t_2}{\partial r_1} &= -\frac{s_2}{(1+\sigma^2)\sigma}\frac{\pm s_a}{\pi} \\ r\frac{\partial t_2}{\partial r_2} &= +\frac{s_1}{(1+\sigma^2)\sigma}\frac{\pm s_b}{\pi} \end{aligned}$$

so that when this mapping is composed with a mapping in cylindrical coordinates form the terms will cancel nicely to remove the removable singularity.

The inverse of the mapping $(t_1, t_2) \rightarrow (r_1, r_2)$ is defined by

$$\zeta = 2t_1 - 1, \quad r = \sqrt{1-\zeta^2} \quad \tan(\theta) = \frac{\pm s_2}{s_1}, \quad s_1 = \frac{r}{1\pm\zeta}\cos\theta, \quad s_2 = \pm\frac{r}{1\pm\zeta}\sin\theta.$$

$$r_1 = \frac{s_1}{s_a} + \frac{1}{2}, \quad r_2 = \frac{s_2}{s_b} + \frac{1}{2}$$

The derivatives are returned as

$$\begin{aligned} -r\frac{\partial r_1}{\partial t_1} &= \frac{\cos(\theta)}{(1\pm\zeta)}\frac{\pm 2}{s_a} \\ -r\frac{\partial r_2}{\partial t_1} &= \frac{\sin(\theta)}{(1\pm\zeta)}\frac{2}{s_b} \\ \frac{1}{r}\frac{\partial r_1}{\partial t_2} &= \frac{\sin(\theta)}{(1\pm\zeta)}\frac{-2\pi}{s_a} \\ \frac{1}{r}\frac{\partial r_2}{\partial t_2} &= \frac{\cos(\theta)}{(1\pm\zeta)}\frac{\pm 2\pi}{s_b} \end{aligned}$$

24 Member functions

24.1 Default Constructor

OrthographicTransform(const real sa_ = 1.,
const real sb_ = 1.,
const Pole pole_ = northPole)

Purpose: The `OrthographicTransform` is used by the `ReparameterizationTransform` to remove a polar singularity.

sa_, sb_ (input) : parameters that specify the dimensions of the plane that is projected onto the sphere in the orthographic transform.

pole (input) : reparameterize the `northPole` or the `southPole`.

24.2 setAngularAxis

int
setAngularAxis(const int & tAxis_)

Purpose: Specify which axis (axis1 or axis2) corresponds to the angular (θ) direction of the mapping that will have an orthographic patch on it. The ϕ direction will be axis1 if tAxis=axis2 or axis2 if tAxis=axis1.

tAxis_ (input) : axis1 (0) or axis2 (1).

24.3 setPole

int
setPole(const Pole & pole_)

Purpose: Specify which pole to reparameterize.

pole (input) : reparameterize the `northPole` or the `southPole`.

24.4 setSize

int
setSize(const int & sa_,
const int & sb_)

Purpose: Specify the size of the orthographic patch.

sa_, sb_ (input) : parameters that specify the dimensions of the plane that is projected onto the sphere in the orthographic transform.

24.5 Class PlaneMapping

This mapping defines a plane or rhombus in three-dimensions.

24.6 Constructor

**PlaneMapping(const real & x1 =0. */, const real & y1 /* =0. */, const real & z1 /* =0.,
const real & x2 =1. */, const real & y2 /* =0. */, const real & z2 /* =0.,
const real & x3 =0. */, const real & y3 /* =1. */, const real & z3 /* =0.)**

Purpose: Default Constructor, define a plane (or rhomboid) by three (non-collinear) points, p1=(x1,y1,z1), p2=(x2,y2,z2), p3=(x3,y3,z3) arranged as:

```
p3------------
  |          |
  |          |
  |          |
  |          |
  -----------
p1           p2
```

24.7 setPoints

**int
setPoints(const real & x1 =0. */, const real & y1 /* =0. */, const real & z1 /* =0.,
const real & x2 =1. */, const real & y2 /* =0. */, const real & z2 /* =0.,
const real & x3 =0. */, const real & y3 /* =1. */, const real & z3 /* =0.)**

Purpose: Set the corners of the plane or rhomboid. The plane (or rhomboid) is defined by three (non-collinear) points, p1=(x1,y1,z1), p2=(x2,y2,z2), p3=(x3,y3,z3) arranged as:

```
p3------------
  |          |
  |          |
  |          |
  |          |
  -----------
p1           p2
```

25 QuadraticMapping: define a quadratic curve or surface.

Use this mapping to define a quadratic curve or surface.

A parabola (curve in 2D) is defined by

$$x_0 = c_{0x} + c_{1x} r_0$$
$$x_1 = a_{00} + a_{10} x_0 + a_{20} x_0^2$$

A 3d paraboloid (surface) is defined by

$$x_0 = c_{0x} + c_{1x} r_0$$
$$x_1 = c_{0y} + c_{1y} r_1$$
$$x_2 = a_{00} + a_{10} x_0 + a_{01} x_1 + a_{20} x_0^2 + a_{11} x_0 x_1 + a_{02} x_1^2$$

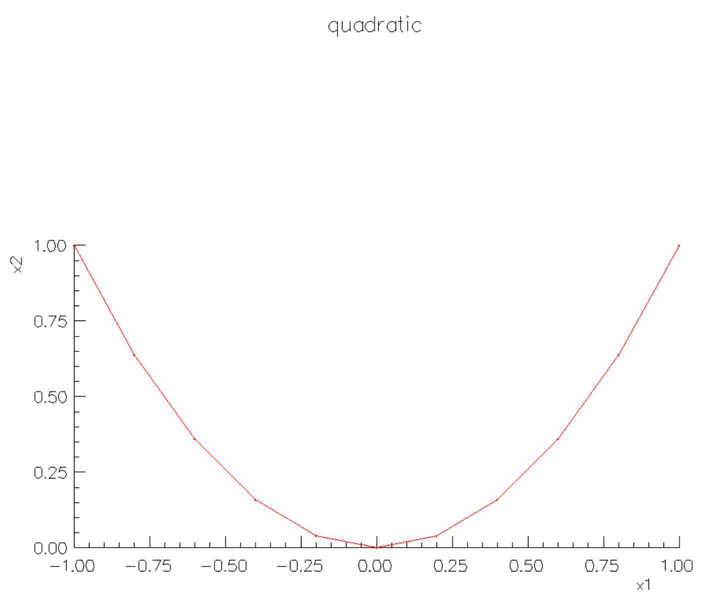
A hyperbola (2d curve) is defined by

$$x_0 = c_{0x} + c_{1x} r_0$$
$$x_1 = \pm (a_{00} + a_{10} x_0 + a_{20} x_0^2)^{1/2}$$

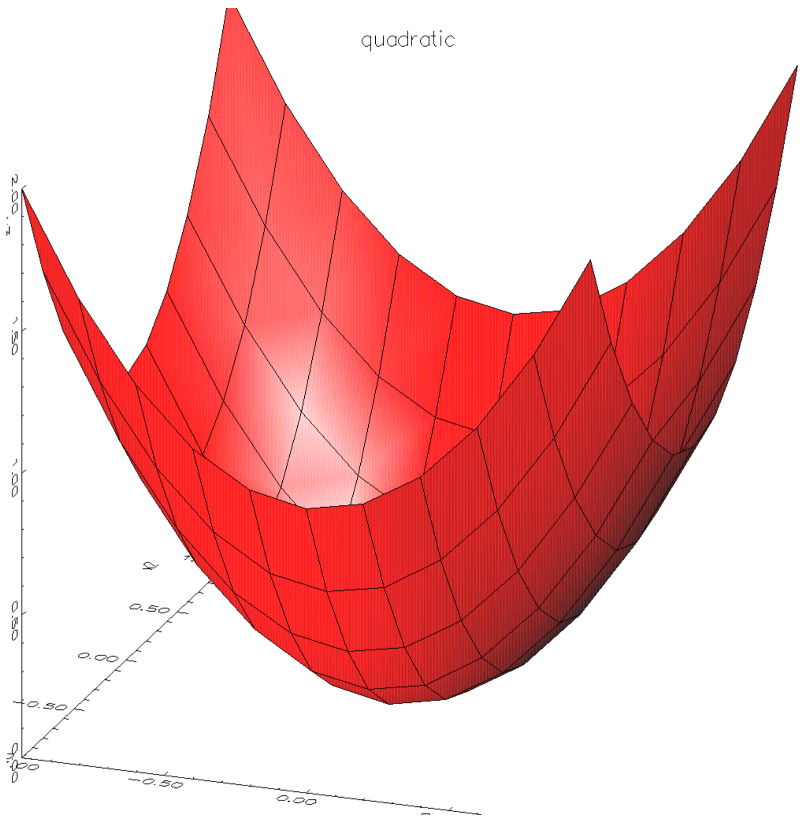
A 3d hyperboloid (surface) is defined by

$$x_0 = c_{0x} + c_{1x} r_0$$
$$x_1 = c_{0y} + c_{1y} r_1$$
$$x_2 = \pm (a_{00} + a_{10} x_0 + a_{01} x_1 + a_{20} x_0^2 + a_{11} x_0 x_1 + a_{02} x_1^2)^{1/2}$$

25.1 Examples



A 2D parabola.



A 3D parabolic surface.

25.2 Constructor

QuadraticMapping()

Description: Define a quadrtic curve or surface (parabola or hyperbola)

25.3 setQuadraticParameters

int
chooseQuadratic(QuadraticOption option,
int rangeDimension_ =2)

Description: Specify the parameters for a quadratic function:

option (input): An option from the enum QuadraticOption: `parabola` or `hyperbola`.

rangeDimension_ (input): 2 or 3

25.4 setParameters

int
setParameters(real c0x,
real c1x,
real c0y,
real c1y,
real a00,
real a10,
real a01,
real a20,
real a11,
real a02,
real signForHyperbola_ = 1.)

Description: Specify the parameters for a quadratic function:

A parabola (curve in 2D) is defined by

$$x_0 = c_{0x} + c_{1x} * r_0$$
$$x_1 = a_{00} + a_{10}x_0 + a_{20}x_0^2$$

A 3d paraboloid (surface) is defined by

$$x_0 = c_{0x} + c_{1x} * r_0$$
$$x_1 = c_{0y} + c_{1y} * r_1$$
$$x_2 = a_{00} + a_{10}x_0 + a_{01}x_1 + a_{20}x_0^2 + a_{11}x_0x_1 + a_{02}x_1^2$$

A hyperbola (2d curve) is defined by

$$x_0 = c_{0x} + c_{1x} * r_0$$
$$x_1 = \pm(a_{00} + a_{10}x_0 + a_{20}x_0^2)^{1/2}$$

A 3d hyperboloid (surface) is defined by

$$x_0 = c_{0x} + c_{1x} * r_0$$
$$x_1 = c_{0y} + c_{1y} * r_1$$
$$x_2 = \pm(a_{00} + a_{10}x_0 + a_{01}x_1 + a_{20}x_0^2 + a_{11}x_0x_1 + a_{02}x_1^2)^{1/2}$$

a00_, a10_,... (input): parameters in above formula.

26 ReductionMapping: create a Mapping from the face or edge of an existing Mapping

26.1 Description

The `ReductionMapping` can be use to make a new Mapping from the face or edge of another Mapping, thus reducing the domain dimension of the original mapping.

In general the new Mapping is defined by fixing one or more of the r-coordinates of the original mapping.

For example if we have a mapping from $\mathbf{R}^3 \rightarrow \mathbf{R}^3$, $\mathbf{x}(r_0, r_1, r_2)$, we can define a new Mapping from $\mathbf{R}^2 \rightarrow \mathbf{R}^3$ by the surface $\mathbf{x}_r(r_0, r_1) = \mathbf{x}(r_0, r_1, r_{2a})$ for some fixed value r_{2a}. We could also define the curve in 3-space by $\mathbf{x}_r(r_0) = \mathbf{x}(r_{0a}, r_0, r_{2a})$ for some fixed values r_{0a} and r_{2a}.

26.2 Constructor

ReductionMapping()

Purpose: Default Constructor

26.3 Constructor

ReductionMapping(Mapping & mapToReduce,
const real & inactiveAxis1Value =0.,
const real & inactiveAxis2Value =-1.,
const real & inactiveAxis3Value =-1.)

Purpose: Create a reduction mapping.

mapToReduce (input): reduce the domain dimension of this mapping.

inactiveAxis1Value (input): if this value is between [0,1] then the r value for axis1 will be fixed to this value and axis1 will become an in-active axis; otherwise axis1 will remain active.

inactiveAxis2Value (input): fix an r value for axis2. See comments for inactiveAxis1Value.

inactiveAxis3Value (input): fix an r value for axis3. See comments for inactiveAxis1Value.

26.4 Constructor

ReductionMapping(Mapping & mapToReduce,
const int & inactiveAxis,
const real & inactiveAxisValue)

Purpose: Create a reduction mapping.

mapToReduce (input): reduce the domain dimension of this mapping.

inactiveAxis (input): This is the inactive axis.

inactiveAxisValue (input): This is the value of the inactive axis in [0,1].

26.5 set

int
set(Mapping & mapToReduce,
const real & inactiveAxis1Value =0. ,
const real & inactiveAxis2Value =-1.,
const real & inactiveAxis3Value =-1.)

Purpose: Set parameters for a reduction mapping.

mapToReduce (input): reduce the domain dimension of this mapping.

inactiveAxis1Value (input): if this value is between [0,1] then the r value for axis1 will be fixed to this value and axis1 will become an in-active axis; otherwise axis1 will remain active.

inactiveAxis2Value (input): fix an r value for axis2. See comments for inactiveAxis1Value.

inactiveAxis3Value (input): fix an r value for axis3. See comments for inactiveAxis1Value.

26.6 set

int
set(Mapping & mapToReduce,
const int & inactiveAxis,
const real & inactiveAxisValue)

Purpose: Set parameters for a reduction mapping.

mapToReduce (input): reduce the domain dimension of this mapping.

inactiveAxis (input): This is the inactive axis.

inactiveAxisValue (input): This is the value of the inactive axis in [0,1].

26.7 setInActiveAxes

int
setInActiveAxes(const real & inactiveAxis1Value =0.,
const real & inactiveAxis2Value =-1.,
const real & inactiveAxis3Value =-1.)

Purpose: Specify the in-active axes.

inactiveAxis1Value (input): if this value is between [0,1] then the r value for axis1 will be fixed to this value and axis1 will become an in-active axis; otherwise axis1 will remain active.

inactiveAxis2Value (input): fix an r value for axis2. See comments for inactiveAxis1Value.

inactiveAxis3Value (input): fix an r value for axis3. See comments for inactiveAxis1Value.

26.8 setInActiveAxes

int
setInActiveAxes(const int & inactiveAxis,
const real & inactiveAxisValue)

Purpose: Set parameters for a reduction mapping.

inactiveAxis (input): This is the inactive axis.

inactiveAxisValue (input): This is the value of the inactive axis in [0,1].

27 ReparameterizationTransform: reparameterize an existing mapping (e.g. remove a polar singularity)

27.1 Description

The `ReparameterizationTransform` can reparameterize a given Mapping in one of the following ways:

Orthographic: remove a polar singularity by using a orthographic projection to define a new patch over the singularity.

Restriction: restrict the parameter space to a sub-rectangle of the original parameter space. Use this, for example, to define a refined patch in an adapative grid.

27.2 Reparameterizing a spherical-polar or cylindrical-polar singularity

The `orthographic` reparameterization can be used to remove a spherical polar singluarity or cylindrical polar singularity by defining a new patch over the singularity.

In order for the `orthographic` reparameterization to be applicable the Mapping to be reparmeterized must have the following properties:

a polar singularity : The mapping must have a polar singularity at $r_1 = 0$ or $r_1 = 1$ and the coordinate direction r_2 must be the angular (θ) variable. (r_3 would be the radial direction). If the mapping has such a singularity then one should indicate this property with the call of the form

```
setTypeOfCoordinateSingularity( side,axis,polarSingularity );
```

can be evaluated in spherical (or cylindrical) coordinates : this property should be set with a call

```
setCoordinateEvaluationType( spherical,TRUE );
```

or

```
setCoordinateEvaluationType( cylindrical,TRUE );
```

A Mapping that can be evaluated in spherical or cylindrical coordinates must define the `map` and `basicInverse` functions to optionally return the derivatives in a special form. For spherical coordinates the derivatives of the mapping are computed as

$$\left(\frac{\partial x_i}{\partial r_1}, \frac{1}{\sin(\phi)} \frac{\partial x_i}{\partial r_2}, \frac{\partial x_i}{\partial r_3} \right)$$

and the derivatives of the inverse mapping as

$$\left(\frac{\partial r_1}{\partial x_i}, \sin(\phi) \frac{\partial r_2}{\partial x_i}, \frac{\partial r_3}{\partial x_i} \right).$$

Here $\phi = \pi r_0$ is the parameter (latitude) for which the spherical singularities occur at $\phi = 0, \pi$. See the implementation of the `SphereMapping` or the `RevolutionMapping` for two examples.

For cylindrical coordinates the derivatives of the mapping are computed as

$$\left(-\rho \frac{\partial x_i}{\partial r_1}, \frac{1}{\rho} \frac{\partial x_i}{\partial r_2}, \frac{\partial x_i}{\partial r_3} \right)$$

and the derivatives of the inverse mapping as

$$\left(\frac{-1}{\rho} \frac{\partial r_1}{\partial x_i}, \rho \frac{\partial r_2}{\partial x_i}, \frac{\partial r_3}{\partial x_i} \right).$$

Here the variable ρ, defined by

$$\zeta = 2r_0 - 1.$$
$$\rho = \sqrt{1 - r_0^2}$$

goes to zero at the singularity. See the implementation of the ellipse in `CrossSectionMapping.C` for an example of cylindrical coordinates.

27.3 Default Constructor

ReparameterizationTransform()

Purpose: Default Constructor The `ReparameterizationTransform` can reparameterize a given Mapping in one of the following ways:

orthographic: Remove a polar singularity by using a orthographic projection to define a new patch over the singularity.

restriction: restrict the parameter space to a sub-rectangle of the original parameter space. Use this, for example, to define a refined patch in an adpative grid.

equidistribution: reparameterize a curve in 2D or 3D so as to equi-distribute a weighted sum of arclength and curvature.

27.4 Constructor(Mapping,ReparameterizationTypes)

ReparameterizationTransform(Mapping & map,
const ReparameterizationTypes type = defaultReparameterization)

Description: Constructor for a Reparameterization.

map (input) : mapping to reparameterize.

type (input) :

27.5 Constructor(MappingRC,ReparameterizationTypes)

ReparameterizationTransform(MappingRC & mapRC,
const ReparameterizationTypes type = defaultReparameterization)

Description: Constructor for a Reparameterization. See the comments in the constructor member function

27.6 constructor(MappingRC,ReparameterizationTypes)

void
constructor(Mapping & map, const ReparameterizationTypes type)

Description: This is a protected routine, used internally. Constructor for a Reparameterization. This constructor will check to see if you are trying to reparameterize a Mapping that is already the same type of reparameterization of another mapping. For example you may be making a sub-mapping (restriction) of a sub-mapping. In this case this constructor will eliminate the multiple restriction operations and replace it by a single restriction. You should then use the scaleBounds member function to define a new restriction. This function will scale the bounds found in map.

27.7 constructorForMultipleReparams

void
constructorForMultipleReparams(ReparameterizationTransform & rtMap)

Description: **This is a protected routine** If you want to reparameterize a mapping that is already Reparameterized then use this constructor. It will replace multiple reparams of the same type with just one reparam

Notes:

27.8 scaleBound

int
scaleBounds(const real ra =0.,
const real rb =1.,
const real sa =0.,
const real sb =1.,
const real ta =0.,
const real tb =1.)

Description: Scale the current bounds for a restriction Mapping. See the documentation for the `RestrictionMapping` for further details.

ra,rb,sa,sb,ta,tb (input):

27.9 getBounds

int
getBounds(real & ra, real & rb, real & sa, real & sb, real & ta, real & tb) const

Description: Get the bounds for a restriction mapping. `RestrictionMapping` for further details.

ra,rb,sa,sb,ta,tb (output):

27.10 setBounds

int
setBounds(const real ra =0.,
const real rb =1.,
const real sa =0.,
const real sb =1.,
const real ta =0.,
const real tb =1.)

Description: Set absolute bounds. See the documentation for the `RestrictionMapping` for further details.

ra,rb,sa,sb,ta,tb (input):

27.10.1 getBoundsForMulitpleReparameterizations

int
getBoundsForMulitpleReparameterizations(real mrBounds[6]) const

Description: Get the bounds for multiple reparameterizations. This routine will usually only be called by the Grid class.

mrBounds (output):

27.10.2 setBoundsForMulitpleReparameterizations

int
setBoundsForMulitpleReparameterizations(real mrBounds[6])

Description: Set the bounds for multiple reparameterizations. This routine will usually only be called by the Grid class.

mrBounds (input):

27.10.3 parameterize

int
setEquidistributionParameters(const real & arcLengthWeight_ /* =1.*/,
const real & curvatureWeight_ /* =0.*/,
const int & numberOfSmooths = 3)

Description: Set the 'arclength' parameterization parameters. The parameterization is chosen to redistribute the points to resolve the arclength and/or the curvature of the curve. By default the curve is parameterized by arclength only. To resolve regions of high curvature choose the recommended values of `arcLengthWeight_=1.` and `curvatureWeight_=1.`.

To determine the parameterization we equidistribute the weight function

$$w(r) = \mathrm{arcLengthWeight}\frac{s(r)}{|s|_\infty} + \mathrm{curvatureWeight}\frac{c(r)}{|c|_\infty}$$

where $s(r)$ is the local arclength and $c(r)$ is the curvature. Note that we normalize s and c by their maximum values.

$$c = |x_{ss}| = \frac{|x_{rr}|}{|x_r|^2}$$

arcLengthWeight_ (input): A weight for arclength. A negative value may give undefined results.

curvatureWeight_ (input): A weight for curvature. A negative value may give undefined results.

numberOfSmooths (input): Number of times to smooth the equidistribution weight function.

28 RestrictionMapping: define a restriction to a sub-rectangle of the unit cube

28.1 Description

The `RestrictionMapping` is a simple mapping that defines a restriction to a sub-rectangle of the unit square or unit cube. This Mapping is used by the `ReparameterizationTransform` where it can be used to define a mesh refinement patch on an adpative grid.

The restriction is a Mapping from `parameter` space $\mathbf{r}$ to `parameter` space $\mathbf{x}$ defined by

$$
\begin{aligned}
x(I,axis1) &= (rb-ra)r(I,axis1)+ra \\
x(I,axis2) &= (sb-sa)r(I,axis2)+sa \\
x(I,axis3) &= (tb-ta)r(I,axis3)+ta
\end{aligned}
$$

28.2 Default Constructor

RestrictionMapping(const real ra_ =0.,
const real rb_ =1.,
const real sa_ =0.,
const real sb_ =1.,
const real ta_ =0.,
const real tb_ =1.,
const int dimension =2 ,
Mapping *restrictedMapping =NULL)

Purpose: Default Constructor The restriction is a Mapping from `parameter` space to `parameter` space defined by

$$
\begin{aligned}
x(I,axis1) &= (rb-ra)r(I,axis1)+ra \\
x(I,axis2) &= (sb-sa)r(I,axis2)+sa \\
x(I,axis3) &= (tb-ta)r(I,axis3)+ta
\end{aligned}
$$

ra_,rb_,sa_,sb_,ta_,tb_ (input): Parameters in the definition of the `RestrictionMapping`.

dimension (input): define the domain and range dimension (which are equal).

restrictedMapping (input) : optionally pass the Mapping being restricted. This Mapping is used to set spaceIsPeriodic.

28.3 scaleBounds

int
scaleBounds(const real ra_ =0.,
const real rb_ =1.,
const real sa_ =0.,
const real sb_ =1.,
const real ta_ =0.,
const real tb_ =1.)

Purpose: Scale the current bounds. Define a sub-rectangle of the current restriction. These parameters apply to the current restriction as if it were the entire unit square or unit cube. For example for the ”r” variable the transformation from old values of (ra,rb) to new values of (ra,rb) is defined by:

$$
\begin{aligned}
rba &= rb-ra \\
rb &= ra+rb\_rba \\
ra &= ra+ra\_rba
\end{aligned}
$$

ra_,rb_,sa_,sb_,ta_,tb_ (input): These parameters define a sub-rectangle of the current restriction.

28.4 getBounds

int
getBounds(real & ra_, real & rb_, real & sa_, real & sb_, real & ta_, real & tb_) const

Description: Get the bounds for a restriction mapping. `RestrictionMapping` for further details.

ra_,rb_,sa_,sb_,ta_,tb_ (output):

28.5 setBounds

int
setBounds(const real ra_ =0.,
const real rb_ =1.,
const real sa_ =0.,
const real sb_ =1.,
const real ta_ =0.,
const real tb_ =1.)

Purpose: Set absolute bounds for the restriction.

ra_,rb_,sa_,sb_,ta_,tb_ (input): Parameters in the definition of the `RestrictionMapping`.

28.6 setSpaceIsPeriodic

int
setSpaceIsPeriodic(int axis, bool trueOrFalse = true)

Description: Indicate whether the space being restricted is periodic. For example if you restrict an AnnulusMapping then you should set periodic1=true since the Annulus is periodic along axis1

29 RevolutionMapping: create a surface or volume of revolution

29.1 Description

The `RevolutionMapping` revolves a two-dimensional mapping (ie. a 2D curve or 2D region) in the plane around a given line to create a three-dimension mapping in three-space. The `RevolutionMapping` can also be used to revolve a curve in 3D about a line to create a surface in 3D.

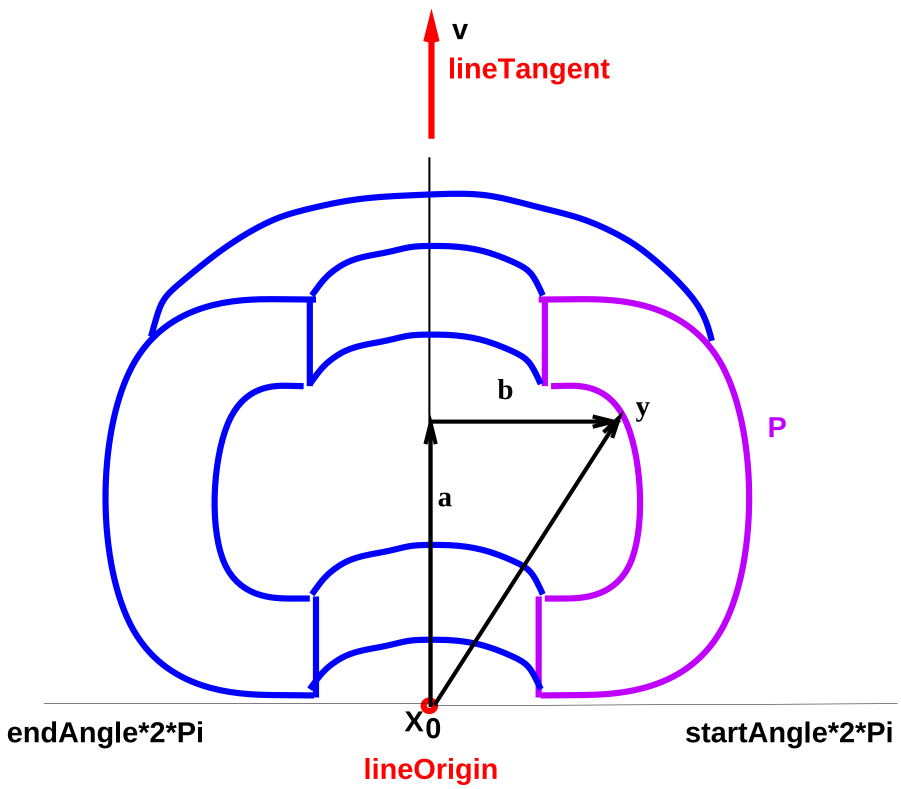


Figure 16: The RevolutionMapping can be used to revolve a 2D mapping about a line through a given angle. It can also be used to revolve a 3D curve.

The revolution mapping is defined in the following manner. (This description applies to the case when the mapping to be revolved is a 2D region. A similar definition applies in the other cases). Let

$$
\begin{aligned}
\mathbf{x}_0 &= \texttt{lineOrigin}(0:2) = \text{a point on the line of revolution} \\
\mathbf{v} &= \texttt{lineTangent}(0:2) = \text{unit tangent to the line of revolution} \\
\mathbf{P} &: \text{ The two dimensional mapping in the plane that we will rotate}
\end{aligned}
$$

We first evaluate the two-dimensional mapping and save in a three dimension vector, $\mathbf{y}$,

$$\mathbf{r}(0:1) \rightarrow (\mathbf{P}(\mathbf{r}), 0) = (y(0:1), 0) \equiv \mathbf{y}$$

Now decompose $\mathbf{y} - \mathbf{x}_0$ into a component parallel and a component orthogonal to the line of revolution:

$$
\begin{aligned}
\mathbf{y} - \mathbf{x}_0 &= \mathbf{a} + \mathbf{b} \\
\mathbf{a} &= (\mathbf{a} \cdot \mathbf{v})\mathbf{v} \qquad \text{component parallel to } \mathbf{v}
\end{aligned}
$$

Then rotate the part orthogonal to the line:

$$\mathbf{x} - \mathbf{x}_0 = \mathbf{a} + R\mathbf{b} \quad \text{where } R \text{ is the rotation matrix}$$

To compute R we determine a vector $\mathbf{c}$ orthogonal to $\mathbf{b}$ and $\mathbf{v}$,

$$\mathbf{c} = \mathbf{v} \times \mathbf{b}$$

Then

$$R\mathbf{b} = \cos(\theta)\mathbf{b} + \sin(\theta)\mathbf{c}$$

where

$$\theta = r(2,I)\delta + \text{startAngle}\, 2\pi, \quad \delta = (\text{endAngle} - \text{startAngle})2\pi$$

In summary the revolution mapping is defined by

$$\begin{aligned}
\mathbf{x} &= \mathbf{a} + \cos(\theta)\mathbf{b} + \sin(\theta)\mathbf{c} + \mathbf{x}_0 \\
\mathbf{a} &= ((\mathbf{y} - \mathbf{x}_0)\cdot \mathbf{v})\mathbf{v} \\
\mathbf{b} &= \mathbf{y} - \mathbf{x}_0 - \mathbf{a} \\
\mathbf{c} &= \mathbf{v} \times \mathbf{b}
\end{aligned}$$

The derivatives of the mapping are defined as

$$\begin{aligned}
\frac{\partial \mathbf{x}}{\partial r_i} &= \frac{\partial \mathbf{a}}{\partial r_i} + \cos(\theta)\frac{\partial \mathbf{b}}{\partial r_i} + \sin(\theta)\frac{\partial \mathbf{c}}{\partial r_i} \qquad \text{for } i = 0,1 \\
\frac{\partial \mathbf{a}}{\partial r_i} &= (\frac{\partial \mathbf{y}}{\partial r_i}\cdot \mathbf{v})\mathbf{v} \\
\frac{\partial \mathbf{b}}{\partial r_i} &= \frac{\partial \mathbf{y}}{\partial r_i} - \frac{\partial \mathbf{a}}{\partial r_i} \\
\frac{\partial \mathbf{c}}{\partial r_i} &= \mathbf{v} \times \frac{\partial \mathbf{b}}{\partial r_i} \\
\frac{\partial \mathbf{x}}{\partial r_2} &= \delta * (-\sin(\theta)\mathbf{b} + \cos(\theta)\mathbf{c})
\end{aligned}$$

29.2 Inverse of the mapping

When the mapping to be revolved is in the x-y plane, the `RevolutionMapping` can be inverted analytically in terms of the inverse of the mapping that is being revolved. (When revolving a 3D curve we do not define a special inverse). Here is how we do the inversion.

Given a value $\mathbf{x}$ we need to determine the corresponding value of $\mathbf{r}$. To do this we can first decide how to rotate the point $\mathbf{x}$ (about the line through $\mathbf{x}_0$ with tangent $\mathbf{v}$) into the x-y plane. This will determine θ and $\mathbf{y}$. Given $\mathbf{y}$ we can invert the two-dimensional mapping to determine (r_0,r_1).

To perform the rotation back to the $x-y$ plane we decompose $\mathbf{x} - \mathbf{x}_0$ into

$$\mathbf{x} - \mathbf{x}_0 = \mathbf{a} + \hat{\mathbf{b}}$$

where $\mathbf{a}$ is the component parallel to $\mathbf{v}$, (the same $\mathbf{a}$ as above),

$$\mathbf{a} = ((\mathbf{x} - \mathbf{x}_0)\cdot \mathbf{v})\mathbf{v}$$

and

$$\hat{\mathbf{b}} = \mathbf{x} - \mathbf{x}_0 - \mathbf{a}.$$

Note that $\hat{\mathbf{b}}$ is not the same as $\mathbf{b}$. Letting

$$\hat{\mathbf{c}} = \pm\mathbf{v} \times \hat{\mathbf{b}}$$

where the correct sign must be chosen, then we can rotate back to the $x-y$ plane with the transformation

$$\mathbf{y} = \mathbf{a} + \cos(-\theta)\hat{\mathbf{b}} + \sin(-\theta)\hat{\mathbf{c}} \tag{15}$$

Since $y_3 = 0$ ($\mathbf{y} = (y_1, y_2, y_3)$) it follows from the third component of this last equation that

$$0 = a_3 + \cos(\theta)\hat{b}_3 - \sin(\theta)\hat{c}_3$$

Now assuming that $a_3 = 0$ (which assumes that the line of rotation is in the $x-y$ plane) then

$$\tan(\theta) = \frac{\hat{b}_3}{\hat{c}_3}$$

Given θ we then know the first two components of $\mathbf{y} = (y_1, y_2, 0)$ from (15). We now determine the inverse of (y_1, y_2) using the `inverseMap` of the two-dimensional mapping,

$$\mathbf{r}(0:1) = \mathbf{P}^{-1}(\mathbf{y}(0:1))$$

29.3 Reparameterized to spherical-like coordinates

If the body of revolution created by this mapping has a spherical polar singularity at one or both ends we may wish to create a new mapping near the pole that does not have a singularity by using an orthographic mapping (created from the `reparameterize` menu in `ogen`). The orthographic transform expects the mapping to be parameterized like a sphere with parameters (ϕ, θ, r). Thus we will want to change the order of the parameters in the above definition of the body of revolution:

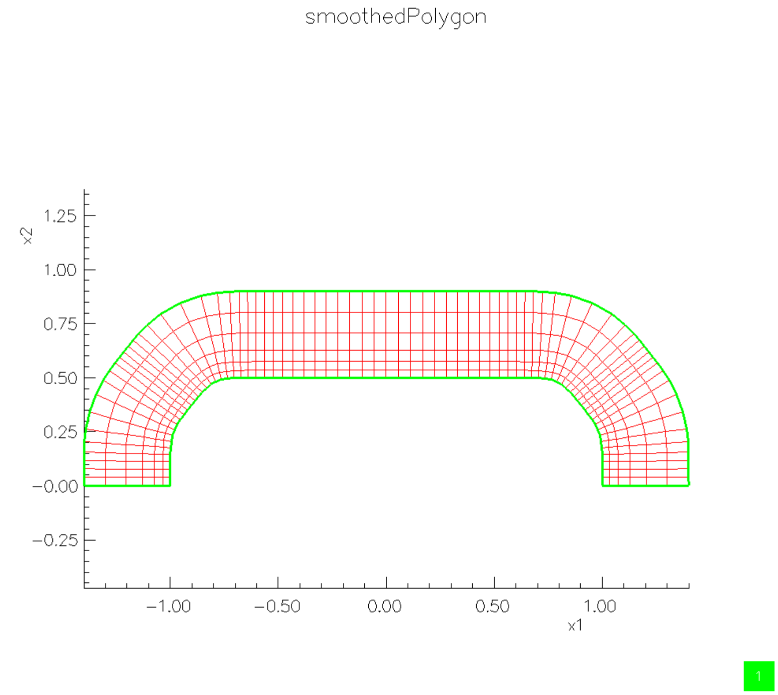
$$\tilde{\mathbf{x}}(r_1, r_2, r_3) = \mathbf{x}(r_1, r_3, r_2) \quad \text{or} \quad \tilde{\mathbf{x}}(r_1, r_2, r_3) = \mathbf{x}(r_3, r_1, r_2)$$

so that the new variable $\tilde{\mathbf{x}}$ will be parameterized like a sphere.

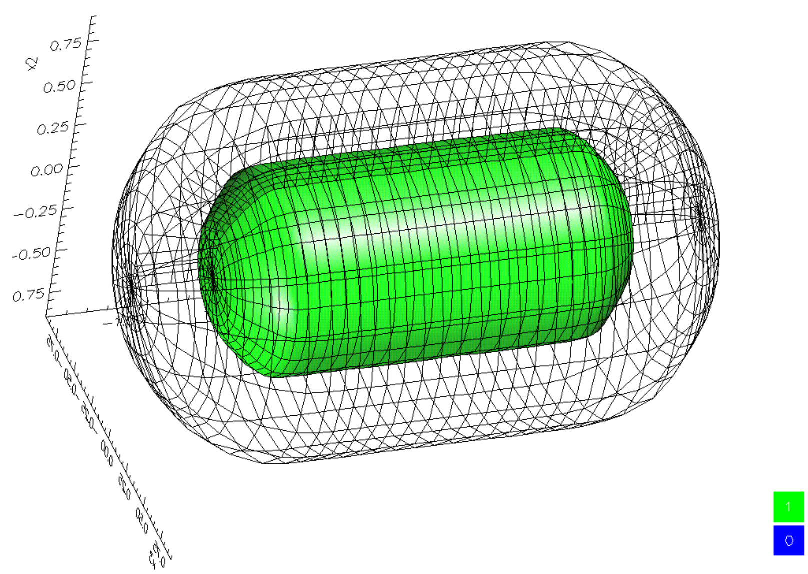
This re-ordering is done automatically if the body of revolution is detected to have a spherical polar type singularity.

29.4 Examples

```
*
* Create a cylindrical body of revolutic
* from a Smoothed Polygon
*
SmoothedPolygon
  vertices
  7
  -1. 0.
  -1. .25
  -.8 .5
  0. .5
  .8 .5
  1. .25
  1. 0.
  n-dist
  fixed normal distance
  .1
  n-dist
  fixed normal distance
  .4
  corners
  specify positions of corners
  -1. 0.
  1. 0
  -1.4 0.
  1.4 0
  t-stretch
  0 5
  .15 10
  .15 10
  0 10
  .15 10
  .15 10
  0 10
exit
body of revolution
  tangent of line to revolve about
  1. 0 0
  boundary conditions
    0 0 -1 -1 1 0
  mappingName
    cylinder
  lines
    45 21 7
* exit
```

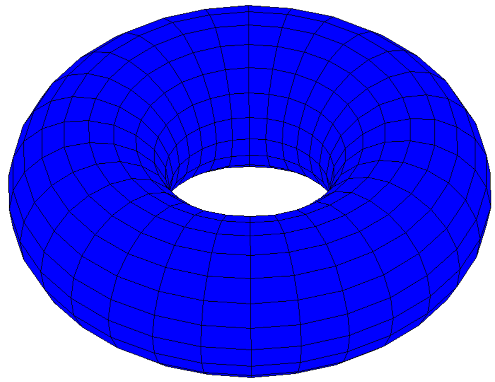


A two-dimensional smoothed polygon.



A body of revolution created by revolving a two-dimensional smoothed polygon.

```
Circle or ellipse
  exit
body of revolution
  choose a point on the line to revolve abou
    -2. 0 0
```



A body of revolution for a torus is created by revolving a circle.

29.5 Constructor

RevolutionMapping()

Purpose: Default Constructor

29.6 Constructor

RevolutionMapping(Mapping & revolutionary_,
const real startAngle_ =0.,
const real endAngle_ =1.,
const RealArray & lineOrigin_ =Overture::nullRealDistributedArray(),
const RealArray & lineTangent_ =Overture::nullRealDistributedArray())

Purpose: This constructor takes a mapping to revolve plus option parameters

revolutionary_ (input) : mapping to revolve.

startAngle_ (input) : starting "angle" (in [0,1]) for the reolution.

endAngle_ (input) : ending "angle" (in [0,1]) for the revolution.

lineOrigin_ (input) : the point of origin for the line of revolution.

lineTangent_ (input) : the tangent to the line of revolution.

29.7 setRevolutionAngle

int
setRevolutionAngle(const real startAngle_ =0.,
const real endAngle_ =1.)

Purpose: Define the angle through which the revolution progresses.

startAngle_ (input) : starting "angle" (in [0,1]) for the revolution.

endAngle_ (input) : ending "angle" (in [0,1]) for the revolution.

29.8 getRevolutionAngle

int
getRevolutionAngle(real & startAngle_,
real & endAngle_)

Purpose: Get the bounding angles in the revolution progresses.

startAngle_ (input) : starting "angle" for the revolution.

endAngle_ (input) : ending "angle" for the revolution.

29.9 setParameterAxes

int
setParameterAxes(const int & revAxis1_, const int & revAxis2_, const int & revAxis3_)

Purpose: Define the parameter axes the mapping. The 2D mapping will be evaluated with `(r(I,revAxis1),-r(I,revAxis2))` while `r(I,revAxis3)` will correspond to the angle of revolution θ. The choice of these variables is normally only important if the body of revolution has a spherical polar singularity at one or both ends and the user wants to remove the singularity using the orthographic projection.(reparameterization option). The orthographic project expects the mapping to parameterized like a sphere with the parameters in the order (ϕ,θ,r).

revAxis1 The axis corresponding to ϕ in a spherical coordinate systems or the axial variable s in cylindrical coordinates. `revAxis1` will normally be 0 (or 1) and correspond to the axial like variable in the 2D mapping that is being revolved.

revAxis2 The axis corresponding to r in a spherical coordinate system. Normally revAxis2=2 so the axial variable appears last.

revAxis3 The axis corresponding to θ in a spherical coordinate system. Normally revAxis3=1.

revAxis1_,revAxis2_,revAxis3_ (input) : A permutation of (0,1,2).

29.10 setRevolutionary

int
setRevolutionary(Mapping & revolutionary_)

Purpose: Define the mapping that will be revolved.

revolutionary_input) : mapping to revolve.

29.11 setLineOfRevolution

int
setLineOfRevolution(const RealArray & lineOrigin_,
const RealArray & lineTangent_)

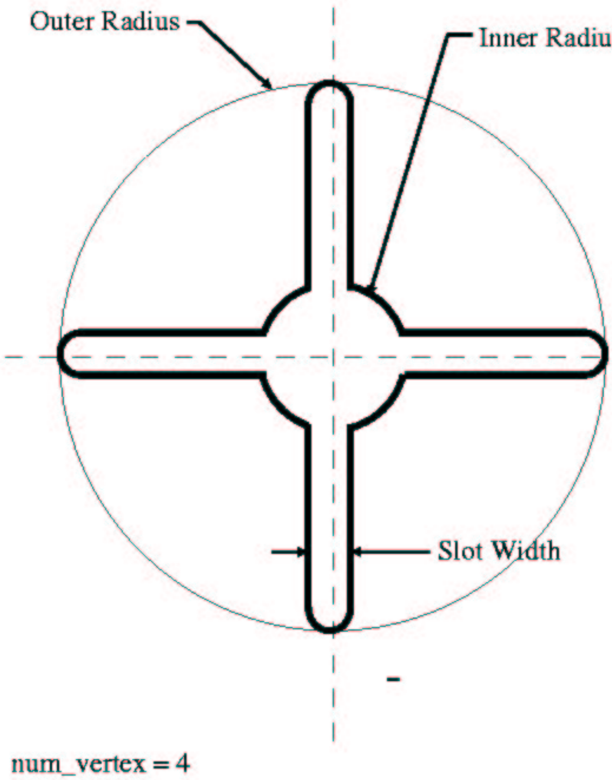
Purpose: Define the point of origin and the tangent of the line of revolution. *For now this point and line must lie in the x-y plane (lineOrigin_(2)==0, lineTangent_(2)==0)

lineOrigin_ (input) : the point of origin for the line of revolution. For now we require lineOrigin_(2)==0.

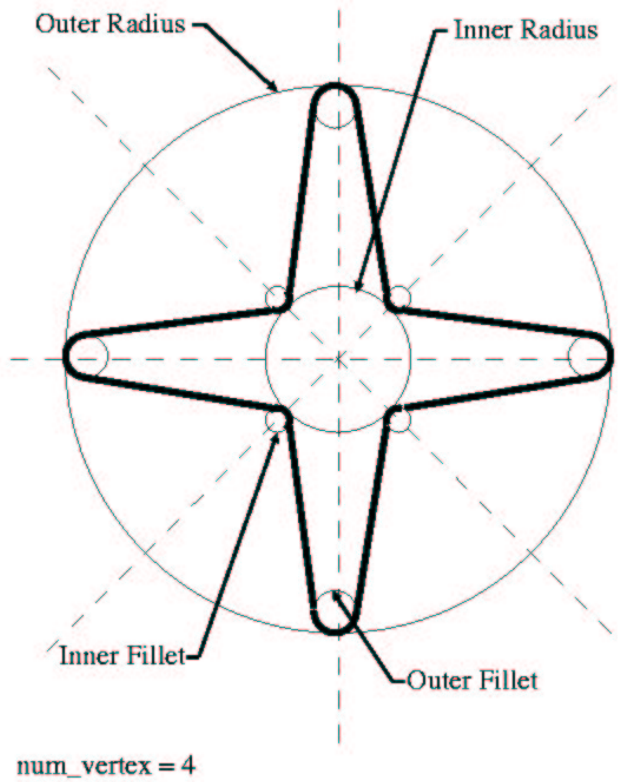
lineTangent_ (input) : the tangent to the line of revolution with lineTangent_(2)==0

30 RocketMapping: create rocket geometry curves

The `RocketMapping` defines a variety of curves related to rockets. The curves defined in this class were originally written by Nathan Crane (as three separate Mapping's), and then subsequently reorganized into a single class by WDH. There are currently 3 cross-section shapes supported, the **slot**, **star** and **circular** shapes. The slot and star shapes are illustrated in the next figure.



The slot shape.



The star cross-section.

30.1 Slot

Overview : The slot option creates by default a sloted grain shaped spline in the z=0 plane. The mapping should be usable in every way as a standard spline. The slot spline mapping will always be periodic, the number and location of the spline knot points are generated automatically according to the slotted grain input parameters. A graphical description of the various parameters can found in the above figure.

Options:

set range dimension : Toggle between a 2D and 3D spline (spline will always lie in a plane, but in 3D that plane can be rotated or shifted to a arbitrary positon.)

shape preserving (toggle) : toggle between shape preserving and tension spline (see standard spline mapping documentation for more info)

set bounding radii : Set the inner and outer bounding radii for the slot grain. slots will just touch a circle of radius outer bounding radius. The slots will intersect a circle of radius inner bounding radius.

set slot width : Set the width of each slot. The slots wil be rounded on the ends by a circle of diameter slot width.

set z value : by default the spline lies in the z=0 plane. Changing the z value moves the spline to some other constant z value plane. This command is a shortcut for the shift operator. This command cannot be used with a 2D sloted grain mapping.

set element size : Set the size of the elements along the spline. The total length of the spline is computed, and the number of lines of grid points is taken as total_length/el_size. the number of spline knot points is taken as the same as the number of lines. Element size and number of lines are mutally exclusive commands.

set number of vertex : Set the number of vertices (number of slots) of the mapping valid values are 2 vertices and up.

30.2 Star

Overview: The star option creates by default a star shaped spline in the z=0 plane. The mapping should be usable in every way as a standard spline. The star spline mapping will always be periodic, the the number and location of the spline knot points are generated automatically according to the star input parameters. A graphical description of the various parameters can found in the above figure.

Options:

set range dimension : Toggle between a 2D and 3D spline (spline will always lie in a plane, but in 3D that plane can be rotated or shifted to a arbitrary positon.)

shape preserving (toggle) : toggle between shape preserving and tension spline (see standard spline mapping documentation for more info)

set bounding radii : set the inner and outer bounding radii for the star. The inside points the star will be circimscribed between the inner and outer radi. The outer points of the star will just touch a circle of radius outer bounding radius. The inner points of the star will just touch a circle of radius inner bounding radius.

set fillet radii : set the inner and outer fillet radii for the star. The fillet radi determine the sharpness of the points of the star. Large fillet radii create a more blun star, while smaller fillet radi create a sharper star. Note that when createing in 3D volume, a sharper pointed star will require a thiner boundry mesh, and thus more elements to mesh than a blunted star.

set z value : by default the star lies in the z=0 plane. Changing the z value moves the star to some other constant z value plane. This command is a shortcut for the shift operator. This command cannot be used with a 2D star.

set element size : Set the size of the elements along the star. The total length of the star spline is computed, and the number of lines of grid points is taken as total_length/el_size. the number of spline knot points is taken as the same as the number of lines. Element size and number of lines are mutally exclusive commands.

set number of vertex : Set the number of vertices (number of arms) of the star valid values are 2 vertices and up. For instance a space shuttle fuel grain is described by a 11 vertex star.

set number of points : explicitly sets the number of knot points for the spline. This command overrides the set element size command.

30.3 circle

Overview : The CircSplineMapping creates by default a circular spline in the z=0 plane. The mapping should be usable in every way as a standard spline. The circular spline mapping will always be periodic, the the number and location of the spline knot points are generated automatically according to the circle parameters. The circualr spline mapping is need when creating rocket cross sections to correctly parameterize the star, anular, and sloted grain portions in a compatible way.

30.4 Member functions

30.4.1 Constructor

RocketMapping(const int & rangeDimension_ =2)

Purpose: Define various cross-sections related to rocket geometries

rangeDimension_ : 2, 3

Author: Nathan Crane, cleaned up by Bill Henshaw.

30.4.2 computePoints

int
computePoints()

Purpose: Compute the spline points.

30.4.3 computeSlotPoints

int
computeSlotPoints()

Purpose: Compute the spline points for a slotted cross-section.

30.4.4 computePoints

int
computeStarPoints()

Purpose: Supply spline points for a star cross-section.

30.4.5 computeCirclePoints

int
computeCirclePoints()

Purpose: Supply spline points for a 2D Circ.

30.4.6 update

int
update(MappingInformation & mapInfo)

Purpose: Interactively create and/or change the spline mapping.

mapInfo (input): Holds a graphics interface to use.

31 SmoothedPolygon

This mapping can be used to create a grid whose one edge is a polgyon with smoothed corners. The grid is created by extending normals from the smoothed polygon.

The smoothed polygon is defined by a sequence of vertices

$$\mathbf{x}_v(i) = (x_v(i), y_v(i))), \quad i = 0, 1, \ldots, n_v - 1$$

The curve is parameterized by a pseudo-arclength s, $0 \le s \le 1$ with the value of s at vertex i defined by the relative distance along the (un-smoothed) polygon:

$$s(i) = \frac{\sum_{j=0}^{i-2} \|\mathbf{x}_v(j+1) - \mathbf{x}_v(j)\|}{\sum_{j=0}^{n_v-2} \|\mathbf{x}_v(j+1) - \mathbf{x}_v(j)\|}$$

The smoothed polygon is defined using the *interval* functions $V_j(s)$ of the StretchMapping class. The interval functions can be used to smoothly transition from one slope to a second slope. For example, the x component of the smoothed polygon is defined as

$$x(s) = \left[s + \sum_{j=0}^{n_v} (V_j(s) - V_j(0)) \right] c_1 + c_0$$

Recall that the interval function V_j is dependent on the three parameters d_j, e_j and f_j. The parameter d_j for V_j is given by

$$d_j = \frac{x_v(j+1) - x_v(j)}{s(j+1) - s(j)} \quad j = 0, 1, \ldots, n_v - 2$$

while

$$f_j = s(j)$$

The value of e_j is specified by the user (default = 40) and determines the sharpness of the curve at the vertex.

A grid is defined from this smooth polygon by extending normals. The length of the normal can be constant or can be made to vary. If r_1 parameterizes the curve in the tangential direction and r_2 in the normal direction then the parameterization of the grid is given by

$$\mathbf{x}(r_1, r_2) = \mathbf{x}(r_1) + r_2 N(r_1) \mathbf{n}(r_1)$$

The function $N(r_1)$ is itself defined in terms of stretching functions.

The user has the option to stretch the grid lines in the tangential direction in order to concentrate grid lines near the vertices. The user may also stretch the grid lines in the normal direction. Of course the grid lines may also be stretched by composing this mapping with a StretchMapping.

Note: Unfortunately the smoothed polygon only matches the corners exponentially close with respect to the `sharpness` parameter. Moreover the higher numbered vertices will have larger errors (cf. the formula above). If you choose small values for the sharpness then the SmoothedPolygon will not match the vertices very well, nor will it be symmetric.

moothedPolygonInclude.tex

31.1 update(MappingInformation &)

The SmoothPolygon Mapping is defined interactively through a graphics interface:

```
GL_GraphicsInterface graphicsInterface;          // create a GL_GraphicsInterface object
MappingInformation mappingInfo;
mappingInfo.graphXInterface=&graphicsInterface;

...
SmoothPolygon poly;
poly.interactiveConstructor( mappingInfo );      // interactively create the smoothed polygon
```

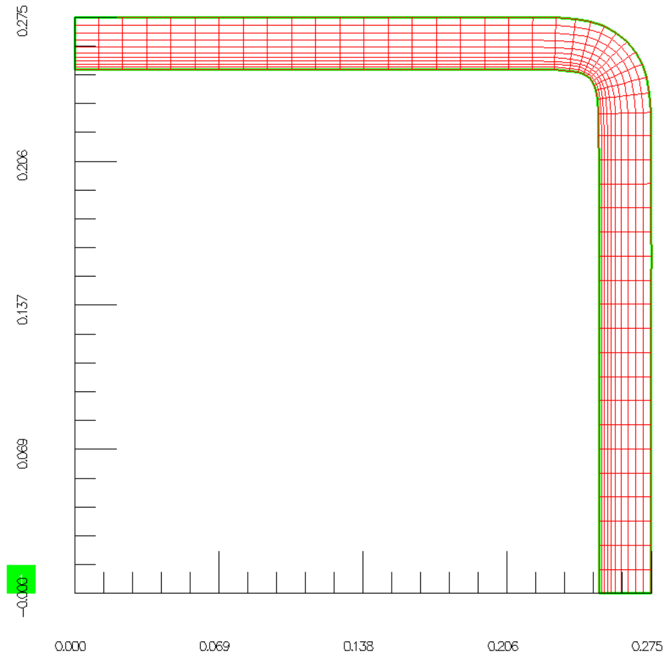
The user must specify the vertices of the polygon. The user may then optionally change various parameters from their default values.

- **sharpness** : Specify how sharp the corners are (exponent). Choose the value for e_j in V_j. Note that if you choose small values for the sharpness then the SmoothedPolygon will not match the vertices very well, nor will it necessarily be symmetric.
- **t-stretch** : Specify stretching in tangent-direction. Specify the values for a_j and b_j for the exponential layer stretching at corner j
- **n-stretch** : Specify stretching in normal-direction. Specify the values for a_i, b_i and c_i for the exponential layer function for stretching in the normal direction and specify the number of layer functions. By default there is one layer function and the grid lines are concentrated near the smoothed polygon with values $a_0 = 1.$, $b_0 = 4.$ and $c_0 = 0.$
- **corners** : Fix the grid corners to specific positions. Use this option to fix the positions of the four corners of the grid. The corners of the grid that lie at a normal distance from the smoothed polygon may not be exactly where you want them because the normal may be slightly different from the line which is perpendicular to the straight line which joins the vertices. This option applies a bi-linear transformation to the entire grid in order to deform the corners to the specified positions.
- **n-dist** : Specify normal distance at vertex(+-epsilon) Choose the normal distance for the grid to extend from the polygon. Optionally the normal distance can be made to vary; a separate normal distance can be given at the position just before vertex i and just after vertex i.
- **curve or area (toggle)** : Change the mapping from defining an area to define a curve (or vice versa). In other words toggle the domain dimension between 1 and 2.
- **isPeriodic**: Specify periodicity array. Indicate whether the grid periodic in the tangential direction. Set this value to 2 if the grid is closed and periodic or to 1 if the grid is not closed but the derivative of the curve is periodic.
- **help** : Print this list
- **exit** : Finished with parameters, construct grid

31.2 Examples

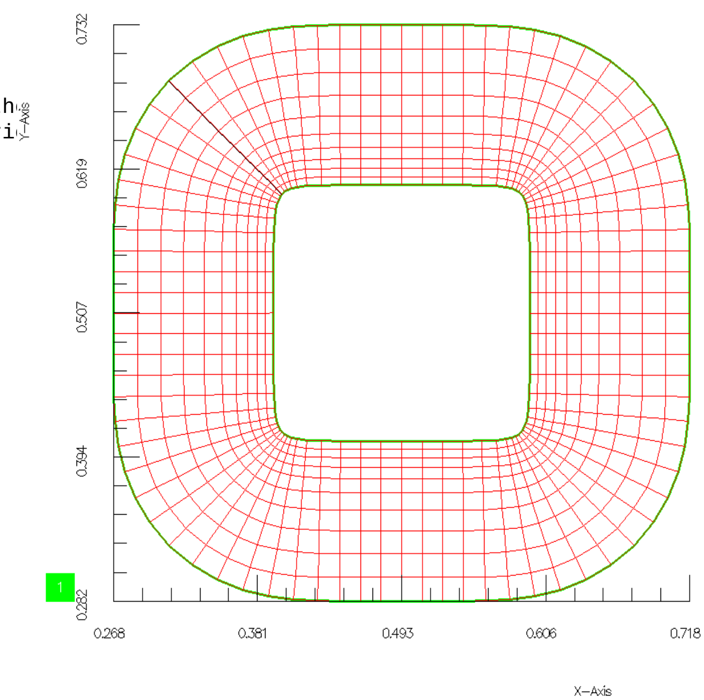
Here are some sample command files that create some SmoothedPolygon mappings. These command files can be read, for example, by the overlapping grid generator `ogen` from within the `create mappings` menu.

```
*
* SmoothedPolygon: example 1
*
SmoothedPolygon
  vertices
    * number of vertices:
    3
    * vertices:
    .25 0.
    .25 .25
    0. .25
  lines
    51 11

```



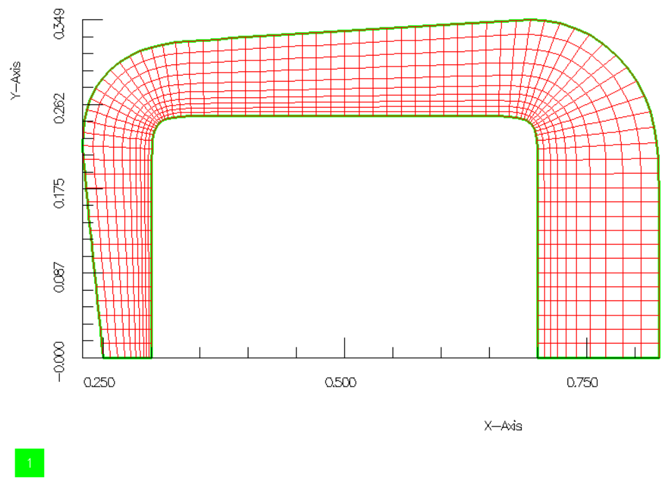
SmoothedPolygon example 1

```
*
* SmoothedPolygon: example 2
*  Note: If first vertex is the sam as th
*  then the mapping is assumed to be peri
*
*
SmoothedPolygon
  vertices
    * number of vertices:
    5
    * vertices:
    .4 .6
    .6 .6
    .6 .4
    .4 .4
    .4 .6
  * specify normal distance
  n-dist
    fixed normal distance
    .125
  lines
    81  11

```



SmoothedPolygon example 2

```
*
* SmoothedPolygon: example 3
*
SmoothedPolygon
  vertices
    * number of vertices:
    4
    * vertices:
    .3  .0
    .3  .25
    .7  .25
    .7  0.
  * specify normal distance
  n-dist
    variable normal distance
    .05 .075  40.
    .075 .1   40.
    .125 .125 40.
  lines
    81  11
```



SmoothedPolygon example 3

32 SphereMapping

This mapping defines a spherical shell or spherical surface in three-dimensions,

$$
\begin{aligned}
\phi &= \pi(\phi_0 + r_1(\phi_1 - \phi_0)) \\
\theta &= 2\pi(\theta_0 + r_2(\theta_1 - \theta_0)) \\
R(r_3) &= (R_0 + r_3(R_1 - R_0)) \\
\mathbf{x}(r_1, r_2, r_3) &= (R\cos(\theta)\sin(\phi) + x_0, R\sin(\theta)\sin(\phi) + y_0, R\cos(\phi) + z_0)
\end{aligned}
$$

This mapping can be inverted analytically with the inverse defined by

$$
\begin{aligned}
& r := \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} \\
& r_1 = [\cos^{-1}((z-z_0)/r) - \pi\phi_0]/(\pi(\phi_1 - \phi_0)) \\
& r_2 = [\mathrm{atan2}(y_0 - y, x_0 - x) + \pi - 2\pi\theta_0]/(2\pi(\theta_1 - \theta_0)) \\
& r_3 = (r - R_0)/(R_1 - R_0)
\end{aligned}
$$

This mapping can have a spherical polar singularity at one or both ends. Either singularity can be removed by creating an orthographic patch over the pole using the `Reparameterization` transform. In order to do this we must be able to evaluate the derivatives of the `SphereMapping` and its inverse in spherical coordinates. This means we compute the derivatives of the mapping as

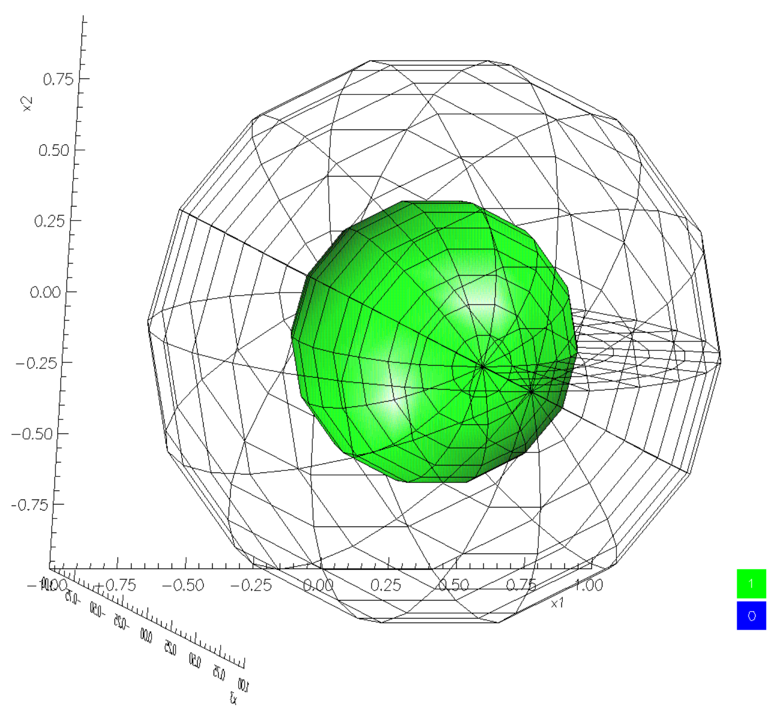
$$
\left(\frac{\partial x_i}{\partial r_1}, \frac{1}{\sin(\phi)} \frac{\partial x_i}{\partial r_2}, \frac{\partial x_i}{\partial r_3} \right)
$$

and the derivatives of the inverse mapping as

$$
\left(\frac{\partial r_1}{\partial x_i}, \sin(\phi) \frac{\partial r_2}{\partial x_i}, \frac{\partial r_3}{\partial x_i} \right).
$$

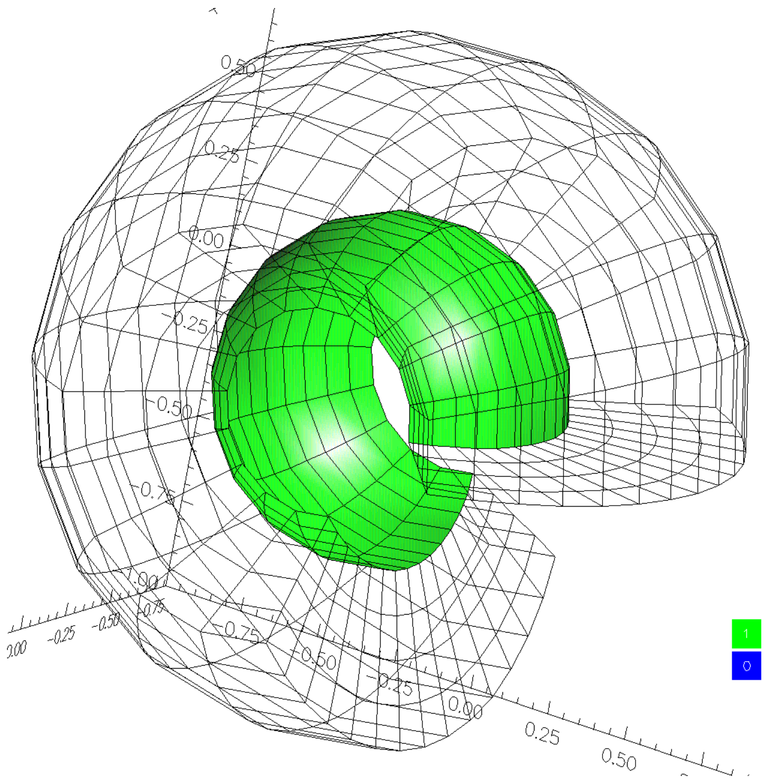
32.1 Examples

```
*
* Make a sphere
*
Sphere
  mappingName
    sphere
  exit
```



A spherical shell built with the `SphereMapping`.

```
Sphere
  bounds on phi (latitude)
    .2 .8
  bounds on theta (longitude)
    0. .8
  exit
```



A partial spherical shell built with the `SphereMapping`.

32.2 Constructor

SphereMapping(const real & innerRadius_ .5,
const real & outerRadius_ 1.,
const real & x0_ .0,
const real & y0_ .0,
const real & z0_ .0,
const real & startTheta_ .0,
const real & endTheta_ 1.,
const real & startPhi_ .0,
const real & endPhi_ 1.)

Description: Define a spherical shell or spherical surface.

innerRadius_,outerRadius_ (input): bounds on the radius.

x0_,y0_,z0_ (input) : center.

startTheta_,endTheta_ (input) : bounds on normalized θ, in the range $[0, 1]$.

startPhi_,endPhi_ (input): bounds on the normalized ϕ, in the range $[0, 1]$.

32.3 setOrigin

int
setOrigin(const real & x0_ =.0,
const real & y0_ =.0,
const real & z0_ =.0)

Description: Specify parameters for the sphere.

x0_,y0_,z0_ (input) : center.

32.4 setPhi

int
setPhi(const real & startPhi_ =.0,
const real & endPhi_ =1.)

Description: Specify parameters for the sphere.

startPhi_,endPhi_ (input): bounds on the normalized ϕ, in the range $[0,1]$.

32.5 setRadii

int
setRadii(const real & innerRadius_ =.5,
const real & outerRadius_ =1.)

Description: Specify parameters for the sphere.

innerRadius_,outerRadius_ (input): bounds on the radius.

32.6 setTheta

int
setTheta(const real & startTheta_ =.0,
const real & endTheta_ =1.)

Description: Specify parameters for the sphere.

startTheta_,endTheta_ (input) : bounds on normalized θ, in the range $[0,1]$.

33 SplineMapping: create a spline curve

Define a cubic spline curve in 1, 2, or 3 space dimensions. The spline curve is chosen to pass through a set of user defined points. Options include

tension : create a spline under tension to remove wiggles, specify a constant tension.

shape preservation : automatic determination of tension factors that vary along the spline so as to create a shape preserving ("monotone") spline.

end conditions : A variety of end conditions for the spline are available:

- **periodic** : The spline can be periodic (choose the periodicity option 'function periodic').
- **derivative periodic** : The derivative of the spline can be periodic (choose the periodicity option 'derivative periodic').
- **monontone parabolic fit** : default BC for the shape preserving spline.
- **first derivative** : user specified first derivatives.
- **second derivative** : user specified second derivatives.

parameterize : by arclength or by weighting the arclength and curvature in order to concentrate grid points near regions with large curvature.

A 2D or 3D spline is parameterized by arclength. A 1D spline is parameterized by the index value of the point. For a spline which is periodic in space, the Mapping will automatically add an extra point if the first point is not equal to the last point.

The SplineMapping uses "**TSPACK**: Tension Spline Curve Fitting Package" by Robert J. Renka; available from Netlib. See the TSPACK documentation and the reference

RENKA, R.J. Interpolatory tension splines with automatic selection of tension factors. SIAM J. Sci. Stat. Comput. **8**, (1987), pp. 393-415.

33.1 Member functions

33.1.1 Constructor

SplineMapping(const int & rangeDimension_ =2)

Purpose: Default Constructor: create a spline curve with the given range dimension. Use this Mapping to create a cubic spline curve in two dimensions. This spline is defined by a set of points (knots), $x(i), y(i)$. The spline is normally parameterized by arclength. The pline can also be parameterized by a weighting of arclength and curvature so that more points are placed in regions with high curvature. For a spline which is periodic in space, the Mapping will automatically add an extra point if the first point is not equal to the last point.

rangeDimension_ : 1,2, 3

The SplineMapping uses '**TSPACK**: Tension Spline Curve Fitting Package' by Robert J. Renka; available from Netlib. See the TSPACK documentation and the reference

RENKA, R.J. Interpolatory tension splines with automatic selection of tension factors. SIAM J. Sci. Stat. Comput. **8**, (1987), pp. 393-415.

33.2 shift

int
shift(const real & shiftx =0.,
const real & shifty =0.,
const real & shiftz /* =0.*/)

Purpose: Shift the SPLINE in space.

33.3 scale

int
scale(const real & scalex =0.,
const real & scaley =0.,
const real & scalez /* =0.*/)

Purpose: Scale the SPLINE in space.

33.4 rotate

int
rotate(const int & axis, const real & theta)

Purpose: Perform a rotation about a given axis. This rotation is applied after any existing transformations. Use the reset function first if you want to remove any existing transformations.

axis (input) : axis to rotate about (0,1,2)

theta (input) : angle in radians to rotate by.

33.4.1 setParameterizationType

int
setParameterizationType(const ParameterizationType & type)

Description: Specify the parameterization for the Spline. With `index` parameterization the knots on the spline are parameterized as being equally spaced. With `arclength` parameterization the knots are parameterized by arclength or a weighted combination of arclength and curvature. With `userDefined` parameterization the user must supply the parameterization through the `setParameterization` function.

type (input) : One of `index` or `arcLength` or `userDefined`.

33.4.2 getParameterization

const realArray &
getParameterization() const

Description: Return the current parameterization.

33.4.3 getNumberOfKnots

int
getNumberOfKnots() const

Purpose: Return the number of knots on the spline.

33.4.4 setParameterization

int
setParameterization(const realArray & s_)

Description: Supply a user defined parameterization. This routine will set the parameterization type to be `userDefined`.

s_ (input) : An increasing sequence of values that are to be used to parameterize the spline points. These values must cover the interval [0,1] which will be the interval defining the mapping. You could add values outside [0,1] to define the behaviour of the spline at "ghost points". The number of points in the array must be equal to the number of points supplied when the `setPoints` function is called.

33.4.5 parameterize

int
parameterize(const real & arcLengthWeight_ /* =1.*/,
const real & curvatureWeight_ /* =0.*/)

Description: Set the 'arclength' parameterization parameters. The parameterization is chosen to redistribute the points to resolve the arclength and/or the curvature of the curve. By default the spline is parameterized by arclength only. To resolve regions of high curvature choose the recommended values of `arcLengthWeight_=1.` and `curvatureWeight_=.5`.

To determine the parameterization we equidistribute the weight function

$$w(r) = 1. + \text{arcLengthWeight}\frac{s(r)}{|s|_\infty} + \text{curvatureWeight}\frac{c(r)}{|c|_\infty}$$

where $s(r)$ is the local arclength and $c(r)$ is the curvature. Note that we normalize s and c by their maximum values.

arcLengthWeight_ (input): A weight for arclength. A negative value may give undefined results.

curvatureWeight_ (input): A weight for curvature. A negative value may give undefined results.

33.4.6 setEndConditions

int
setEndConditions(const EndCondition & condition,
const RealArray & endValues =Overture::nullRealDistributedArray())

Description: Specify end conditions for the spline

condition (input) : Specify an end condition.

monontone parabolic fit : default BC for the shape preserving spline.

first derivative : user specified first derivatives.

second derivative : user specified second derivatives.

endValues (input) : if `condition==firstDerivative` (or `condition==secondDerivative`) then endValues(0:1,0:r-1) should hold the values for the first (or second) derivatives of the spline at the start and end. Here r=rangeDimension.

33.4.7 setPoints

int
setPoints(const realArray & x)

Purpose: Supply spline points for a 1D curve.

x (input) : array of spline knots. The spline is parameterized by a NORMALIZED index, i/(number of points -1), i=0,1,...

33.4.8 setPoints

int
setPoints(const realArray & x, const realArray & y)

Purpose: Supply spline points for a 2D curve. Use the points (x(i),y(i)) i=x.getBase(0),..,x.getBound(0)

x,y (input) : array of spline knots.

33.4.9 setPoints

int
setPoints(const realArray & x, const realArray & y, const realArray & z)

Purpose: Supply spline points for a 3D curve. Use the points (x(i),y(i),z(i)) i=x.getBase(0),..,x.getBound(0)

x,y,z (input) : array of spline knots.

33.4.10 setShapePreserving

int
setShapePreserving(const bool trueOrFalse = TRUE)

Description: Create a shape preserving (monotone) spline or not

trueOrFalse (input) : if TRUE, create a spline that preserves the shape. For a one dimensional curve the shape preserving spline will attempt to remain montone where the knots ar montone. See the comments with TSPACK for further details.

33.4.11 setTension

int
setTension(const real & tensionFactor)

Description: Specify a constant tension factor. Specifying this value will turn off the shape preseeving feature.

tensionFactor (input): A value from 0. to 85. A value of 0. corresponds to no tension.

33.4.12 setDomainInterval

int
setDomainInterval(const real & rStart_ =0.,
const real & rEnd_ =1.)

Description: Restrict the domain of the spline. By default the spline is parameterized on the interval [0,1]. You may choose a sub-section of the spline by choosing a new interval [rStart,rEnd]. For periodic splines the interval may lie in [-1,2] so the sub-section can cross the branch cut. You may even choose rEnd¡rStart to reverse the order of the parameterization.

rStart_,rEnd_ (input) : define the new interval.

33.4.13 getDomainInterval

int
getDomainInterval(real & rStart_, real & rEnd_) const

Description: Get the current domain interval.

rStart_,rEnd_ (output) : the current domain interval.

33.4.14 setIsPeriodic

void
setIsPeriodic(const int axis, const periodicType isPeriodic0)

Description:

axis (input): axis = (0,1,2) (or axis = (axis1,axis2,axis3)) with $axis < domainDimension$.

Notes: This routine has some side effects. It will change the boundaryConditions to be consistent with the periodicity (if necessary).

33.4.15 useOldSpline

int
useOldSpline(const bool & trueOrFalse =TRUE)

Description: Use the old spline routines from FMM, Forsythe Malcolm and Moler. This is for backward compatability.

trueOrFalse (input) : If TRUE Use the old spline from FMM, otherwise use the tension splines.

33.4.16 map

void
map(const realArray & r, realArray & x, realArray & xr, MappingParameters & params)

Purpose: Evaluate the spline and/or derivatives.

33.4.17 update

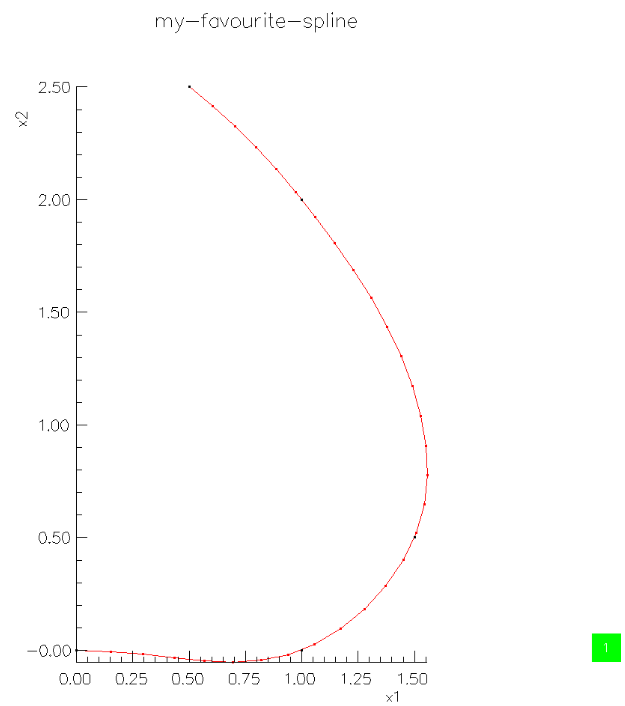
int
update(MappingInformation & mapInfo)

Purpose: Interactively create and/or change the spline mapping.

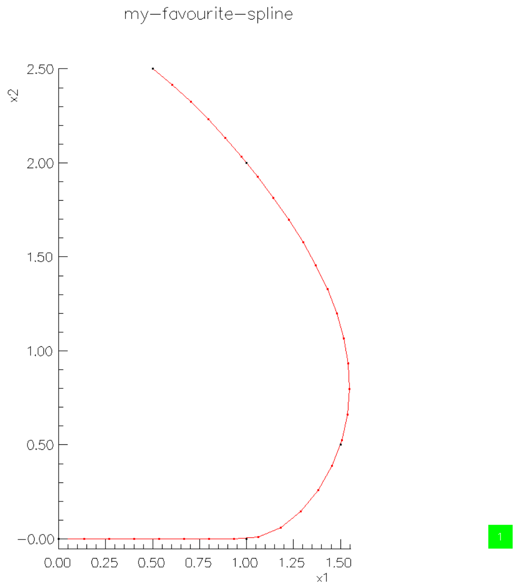
mapInfo (input): Holds a graphics interface to use.

33.5 Examples

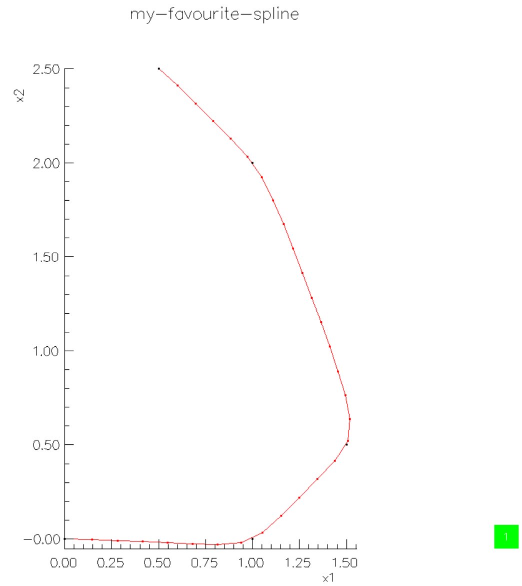
```
*
* Make a 2D spline curve
*
spline
  enter spline points
    * first enter the number of points
    5
    * here are the points (x,y)
    0. 0.
    1. 0.
    1.5 .5
    1. 2.
    .5 2.5
  mappingName
    my-favourite-spline
```



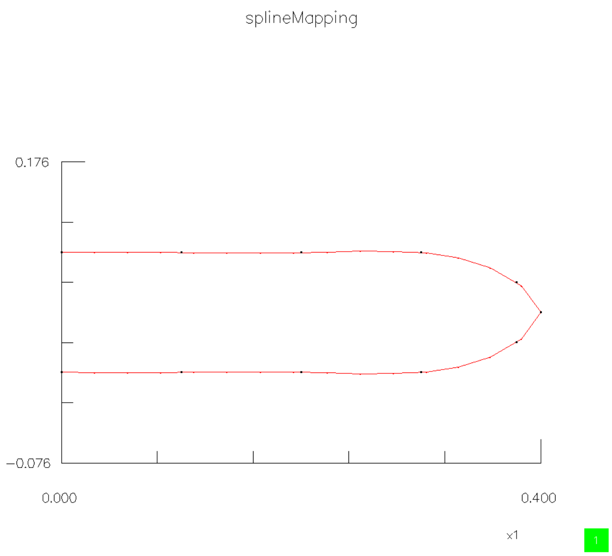
A spline curve in 2D. No Tension.



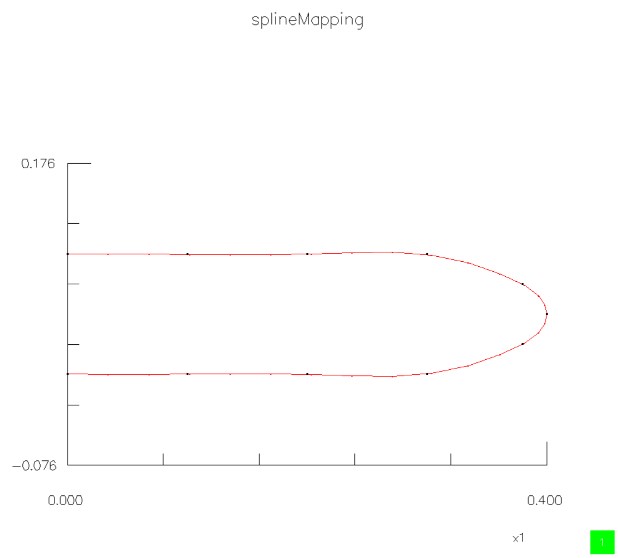
Spline curve with shape preserving option.



Spline curve with tension=20.



Spline curve with default arclength parameterization.



Spline curve with `curvatureWeight=1` so that more points are put where the curvature is large.

34 SquareMapping (rectangles too)

This mapping defines a square or rectangle in two-dimensions

$$\mathbf{x}(r_1,r_2) = (x_a + r_1(x_b - x_a), y_a + r_2(y_b - y_a))$$

34.1 Constructor

SquareMapping(const real xa_, const real xb_, const real ya_, const real yb_)

Purpose: Build a mapping for a square with given bounds.

xa_, xb_, ya_, yb_ (input) : The square is [xa_,xb_]$\times$[ya_,yb_].

34.2 getVertices

real
getVertices(real & xa_ , real & xb_ , real & ya_ , real & yb_) const

Description: return the vertices of the square.

xa_, xb_, ya_, yb_ (output) : The square is [xa_,xb_]$\times$[ya_,yb_].

Return value: is the z-level

34.3 setVertices

void
setVertices(const real xa_ =0.,
const real xb_ =1.,
const real ya_ =0.,
const real yb_ =1.,
const real z_ =0.)

Purpose: Build a mapping for a square with given corners.

xa_, xb_, ya_, yb_ (input) : The square is [xa_,xb_]$\times$[ya_,yb_].

z_ : z level if the rangeDimension is 3.

35 StretchMapping: create 1D stretching functions

The StretchMapping class, derived from the Mapping Class can be used to define one-dimensional “stretching” functions. These functions are often used to stretch grid lines on existing Mappings. These functions can also be used as a blending function for the `TFIMapping`.

There are three types of stretching functions:

35.1 Inverse hyperbolic tangent stretching function

This stretching function is a one-dimensional map from r into x defined (in an inverse fashion) by

$$r = R(x) = \Big[x + \sum_{i=1}^{nu}(U_i(x) - U_i(0)) + \sum_{j=1}^{nv}(V_j(x) - V_j(0))\Big] \times \mathrm{scale} + \mathrm{origin}$$

where $U_i(x)$ is a “layer” function

$$U_i(x) = \frac{a_i}{2}\tanh b_i(x - c_i)$$

and $V_i(x)$ is an “interval” function

$$V_j(x) = \frac{d_j - 1}{2}\log\left(\frac{\cosh e_j(x - f_j)}{\cosh e_j(x - f_{j+1})}\right)\frac{1}{2e_j}$$

The stretching mapping is often used to stretch grid points in parameter space. The functions U_i are used to concentrate grid points in at a point while the functions V_j are used to transition from one grid spacing to another. When the mapping is invertible a spline can be fitted to the inverse to be used as an initial guess for Netwon. Usually only 1-3 Netwon iterations are needed.

Here the terms scale and origin are normalization factors determined so that $R(0) = 0$ and $R(1) = 1$. The remaining parameters are input by the user and have the following constraints:

$$\begin{aligned}
b_j &> 0,\ \ j = 1,..,n_u, \\
0 \le c_j &\le 1,\ \ j = 1,..,n_u, \\
e_j &> 0,\ \ j = 1,..,n_v, \\
f_1 \le 1,\ \ f_{n_v} &\ge 0,\ \ \le f_j \le 1, \quad j = 2,..,n_v - 1, \quad \text{and} \\
f_1 < f_2 < f_3 &< \ ... < f_{n_v}.
\end{aligned}$$

The function $U_i(x)$ is a hyperbolic tangent that is centered at $x = c_i$ and asymptotes to $-a_i/2$ or $a_i/2$ (see Figure 18). As b_i tends to infinity, the function U tends toward a step function.

The function $V_j(x)$ (which is the integral of the difference between two layer functions) is a smoothed-out ramp function with transitions at f_j and f_{j+1} (see Figure 18). The slope of the ramp is d_{j-1}. Thus d_j indicates the relative slope of the ramp compared to the linear term “x,” which appears in $R(x)$. That is, if $d_j = 2$, then the slope of $R(x)$ between f_j and f_{j+1} will be approximately twice the slope of the region where the linear term is dominant. A sloped region can be made to extend past $x = 0$ or $x = 1$ (so that $x = 0$ or $x = 1$ is in the middle of the sloped region) by choosing $f_1 < 0$ or $f_{n_v+1} > 1$. A reasonable value might be $f_1 = -.5$ or $f_{n_v+1} = 1.5$. Note that when a grid is periodic in the r- direction, the functions $U_i(x)$ and $V_j(x)$ are replaced by functions $U_i^p(x)$ and $V_j^p(x)$, respectively, which are given by

$$U_i^p(x) = \sum_{k=-\infty}^{+\infty} U_i(t+k), \quad V_j^p(x) = \sum_{k=-\infty}^{+\infty} V_j(t+k).$$

These functions are not really periodic, but their derivatives with respect to x are periodic with period 1.

The following remarks may prove useful in making choices for the parameters $a_i, ..., f_i$. Below, the variable r typically refers to a uniform grid, while x refers to a grid that has been stretched so that points are clustered in certain locations on the x axis. The clustering of points can be done in two ways. Using the $U_i(x)$ functions (tanh's), the point spacing can be made to decrease exponentially to a minimum spacing at c_i. The value of b_i determines how small the spacing can get. Roughly speaking, a value of $b_i = 10.0$ means the spacing will be about 10 times smaller at the center of the layer than in the unstretched parts of the grid. The relative number of points in this stretched region is proportional to a_i. The linear term x appearing in the definition of $R(x)$ has a weight of one (1), so if there is only one term $U_i(x)$, the relative number of points in the layer is

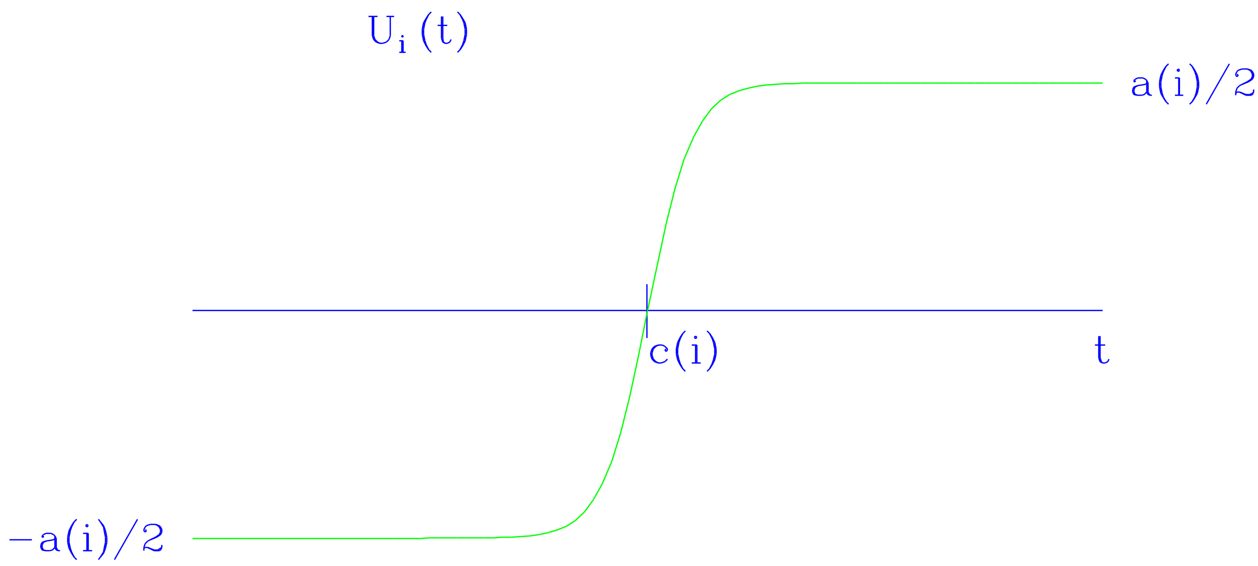


Figure 17: The ‘layer’ function $U(t)$ for concentrating grid lines at a point. The grid spacing is smaller where the slope is larger.

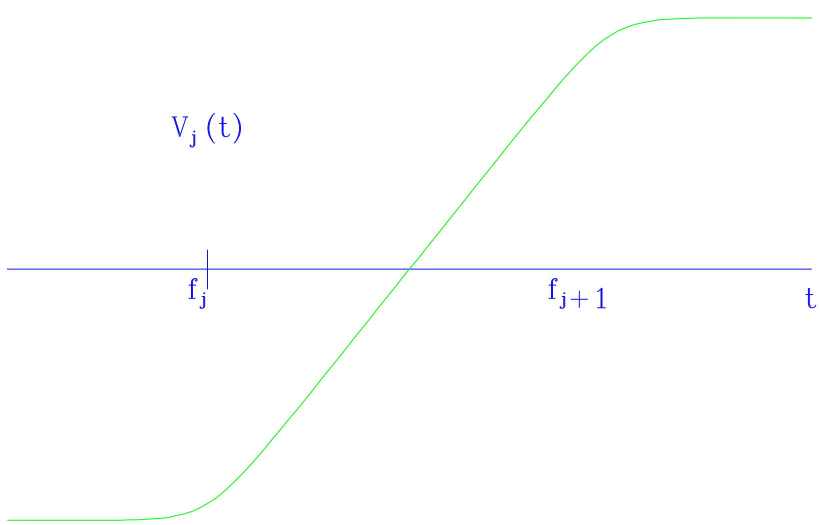


Figure 18: The ‘ramp’ function $V(t)$ for changing from one grid spacing to another. The grid spacing is smaller where the slope is larger.

essentially $a_1/(1+a_1)$. Thus, if $n_u = 1$, ($n_v = 0$), and $a_1 = 1$, then half the points will be in the stretched layer. For two layers, the relative number of points in layer i ($i = 1$ or $i = 2$) is $a_i/(1+a_1+a_2)$. The functions $V_j(x)$ allow you to have intervals where the grid point spacing is relatively smaller or larger than the grid spacing in the region where the linear term x is dominant. In each interval, the grid spacing is nearly constant, except near the transition points f_i and f_{i+1}. The parameter d_i denotes the relative grid spacing in each interval. For example, to make the grid spacing twice as fine for t between 0.25 and 0.5, you would specify $f_1 = 0.25$, $f_2 = 0.5$, and $d_1 = 2$. As another example, to make the spacing 5 times smaller for x between 0 and 0.5, you could say $f_1 = -0.5$, $f_2 = 0.5$, and $d_1 = 5$. Assigning the first transition point a value less than zero, $f_1 = -0.5$, means that $x = 0$ will be in the middle of the interval where the spacing will be 5 times smaller. (If instead $f_1 = 0$, then near $t = 0$ the spacing would be in transition to the default relative grid spacing of 1). The parameters e_i denote how rapid the transition is from one spacing to another. A reasonable value for e_i might be 10.0 or 20.0.

35.2 Hyperbolic tangent stretching function

This function is defined as

$$x(r) = \{a_0 + a_r r + a_1 \tanh(b_1(r-c_1)) + origin\}\, scale$$

If the function is normalized (optional) then origin and scale are chosen to that $x(0) = 0$ and $x(1) = 1$. Note that a_1 will normally be negative in order to concentrate lines near $r = c_1$. To be invertible one should choose $a_r > -a_1 b_1$ (a sufficient but not necessary condition).

35.3 Exponential stretching function

This function is defined as

$$x(r) = \{a_0 + a_r r + a_1 \exp(b_1(r-c_1)) + origin\}\, scale$$

If the function is normalized (optional) then origin and scale are chosen to that $x(0) = 0$ and $x(1) = 1$.

35.4 Exponential blending function

This function is defined as

$$x(r) = \begin{cases} 1 & \frac{3}{4} \le s \le 1 \\ \left[1 + \exp\left(-\frac{\sqrt{3}}{4}\frac{2s-1}{(s-\frac{1}{4})(\frac{3}{4}-s)}\right)\right]^{-1} & \frac{1}{4} < s < \frac{3}{4} \\ 0 & 0 \le s \le \frac{1}{4} \end{cases}$$

This function is used by the `FilletMapping` in order to make a smooth curve in the region where two curves intersect.

35.5 Member function descriptions

35.5.1 Constructor

StretchMapping(const StretchingType & stretchingType_)

Purpose: Construct a function with the given stretching type, one of

- **inverseHyperbolicTangent** : the most commonly used stretching function defined in an inverse way as a combination of hyperbolic tangents and logarithms of hyperbolic cosines.
- **hyperbolicTangent** : hyperbolic tangent stretching.
- **exponential** : exponential stretching.
- **exponentialBlend** : a C^∞ blending function that is exactly 0 for $r < \frac{1}{4}$ and exactly 1 for $r > \frac{3}{4}$.

stretchingType_ (input):

35.5.2 Constructor

StretchMapping(const int numberOfLayers_,
const int numberOfIntervals_)

Purpose: Construct an `inverseHyperbolicTangent` stretching function.

numberOfLayers_ (input): number of layers.

numberOfIntervals_ (input): number of intervals.

35.5.3 setStretchingType

int
setStretchingType(const StretchingType & stretchingType_)

Description: Set the stretching type, one of

- **inverseHyperbolicTangent** : the most commonly used stretching function defined in an inverse way as a combination of hyperbolic tangents and logarithms of hyperbolic cosines.
- **hyperbolicTangent** : hyperbolic tangent stretching.
- **exponential** : exponential stretching.
- **exponentialBlend** : a C^∞ blending function that is exactly 0 for $r < \frac{1}{4}$ and exactly 1 for $r > \frac{3}{4}$.

stretchingType_ (input):

35.5.4 setNumberOfLayers

int
setNumberOfLayers(const int numberOfLayers_)

Description: Set the number of layer (tanh) functions in the `inverseHyperbolicTangent` stretching function.

numberOfLayers_ (input):

Return value: 0 on success, 1 if the stretching type has not been set to `inverseHyperbolicTangent` in which case no changes are made.

35.5.5 setNumberOIntervals

int
setNumberOfIntervals(const int numberOfIntervals_)

Description: Set the number of interval (log(cosh)) functions in the `inverseHyperbolicTangent` stretching function.

numberOfIntervals_ (input):

Return value: 0 on success, 1 if the stretching type has not been set to `inverseHyperbolicTangent` in which case no changes are made.

35.5.6 setNumberOfSplinePoints

int
setNumberOfSplinePoints(const int numberOfSplinePoints0)

Description: Set the number of interval (log(cosh)) functions in the `inverseHyperbolicTangent` stretching function.

numberOfIntervals_ (input):

Return value: 0 on success, 1 if the stretching type has not been set to `inverseHyperbolicTangent` in which case no changes are made.

35.5.7 setLayerParameters

int
setLayerParameters(const int index, const real a, const real b, const real c)

Description: Set parameters for the interval (log(cosh)) function numbered `index`.

a,b,c (input):

Return value: 0 on success, 1 if the stretching type has not been set to `inverseHyperbolicTangent` in which case no changes are made.

35.5.8 setIntervalParameters

int
setIntervalParameters(const int index, const real d, const real e, const real f)

Description: Set parameters for the interval (log(cosh)) function numbered `index`.

d,e,f (input):

Return value: 0 on success, 1 if the stretching type has not been set to `inverseHyperbolicTangent` in which case no changes are made.

35.5.9 setEndPoints

int
setEndPoints(const real rmin, const real rmax)

Description: Set the end points for the `inverseHyperbolicTangent` stretching function.

rmin,rmax (input):

Return value: 0 on success, 1 if the stretching type has not been set to `inverseHyperbolicTangent` in which case no changes are made.

35.5.10 setIsNormalized

int
setIsNormalized(const bool & trueOrFalse =TRUE)

Description: Indicate whether the stretching function should be normalized to go from 0 to 1.

trueOrFalse (input): if TRUE the function is normalized.

35.5.11 setScaleParameters

int
setScaleParameters(const real origin_, const real scale_)

Description: Set the origin and scale parameters for the `inverseHyperbolicTangent` stretching function.

origin_, scale_ (input):

Return value: 0 on success, 1 if the stretching type has not been set to `inverseHyperbolicTangent` in which case no changes are made.

35.5.12 setIsPeriodic

int
setIsPeriodic(const int trueOrFalse)

Description: Define the periodicity of the function, only applies to the `inverseHyperbolicTangent` stretching function.

trueOrFalse (input): TRUE or FALSE.

Return value: 0 on success, 1 if the stretching type has not been set to `inverseHyperbolicTangent` in which case no changes are made.

35.5.13 setHyperbolicTangentParameters

int
setHyperbolicTangentParameters(const real & a0_,
const real & ar_,
const real & a1_,
const real & b1_,
const real & c1_)

Description: Set the parameters for the `hyperbolicTangent` stretching function.

a0_,ar_,a1_,b1_,c1_, (input):

Return value: 0 on success, 1 if the stretching type has not been set to `hyperbolicTangent` in which case no changes are made.

35.5.14 setExponentialParameters

int
setExponentialParameters(const real & a0_,
const real & ar_,
const real & a1_,
const real & b1_,
const real & c1_)

Description: Set the parameters for the `exponential` stretching function.

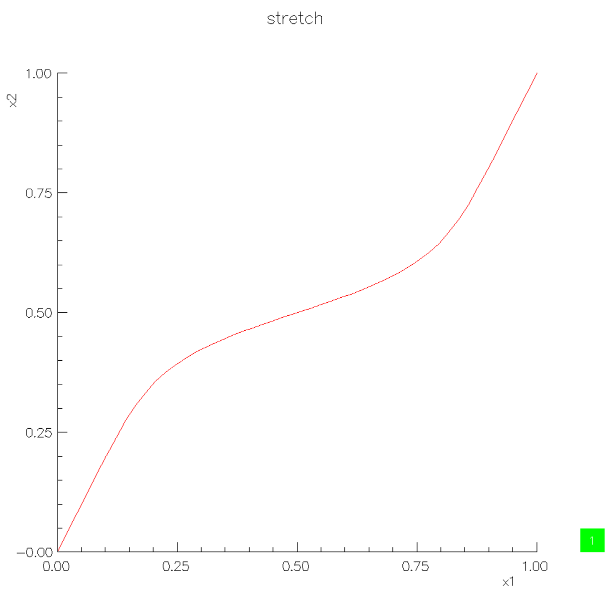
a0_,a1_,b1_,c1_, (input):

Return value: 0 on success, 1 if the stretching type has not been set to `exponential` in which case no changes are made.

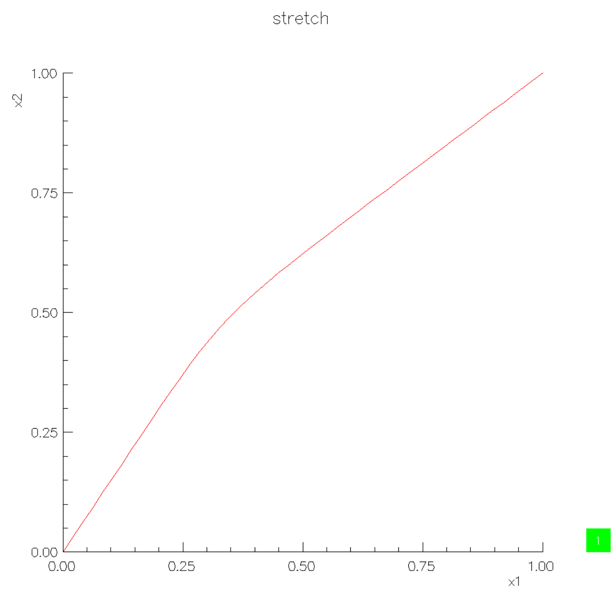
35.6 Examples

Here is an example of the use of the `StretchMapping` class.

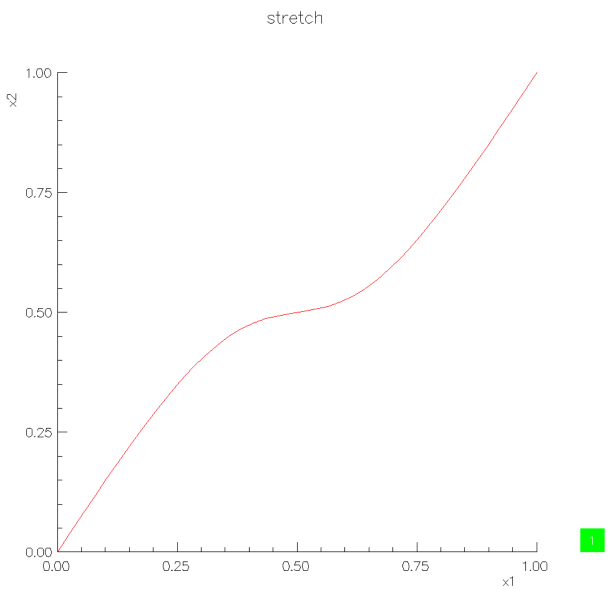
```
#include "Stretch.h"

void main()
{
  const int axis1 = 0;
  const int axis2 = 1;
  const int axis3 = 2;
  realArray r(1,3);
  realArray t(1,3);
  realArray tr(1,3,3);

  StretchMapping stretch1( 2, 0 );                    // two layers, zero intervals
  stretch1.setLayerParameters( 0, 1., 10., .25 );    // set layer 0, a,b,c
  stretch1.setLayerParameters( 1, 1., 10., .75 );    // set layer 1, a,b,c
  stretch1.setIsPeriodic(FALSE);                      // default is FALSE

  r(0,axis1)=.5;
  stretch1.map( r,t,tr );                             // evaluate


  StretchMapping stretch2( 0, 1 );                    // zero layers, one interval
  stretch2.setIntervalParameters( 0, 5., 20., .25 ); // spacing is smaller
  stretch2.setIntervalParameters( 1, 0.,  0., .75 ); // between .25 and .75
  stretch2.setIsPeriodic(FALSE);                      // default is FALSE

  r(0,axis1)=.25;
  stretch2.map( r,t,tr );                             // evaluate

}
```



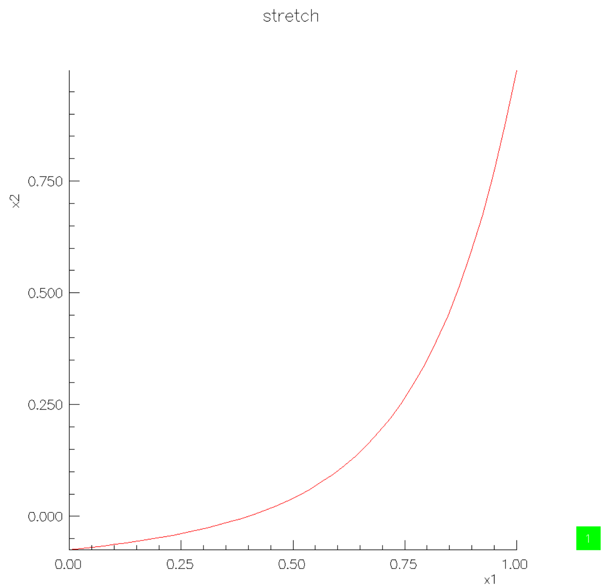
Stretching function: inverseHyperbolicTangent, 1 layer, $a_0 = 1.$, $b_0 = 10.$, $c_0 = .5$. This function will concentrate grid points near $r = .5$



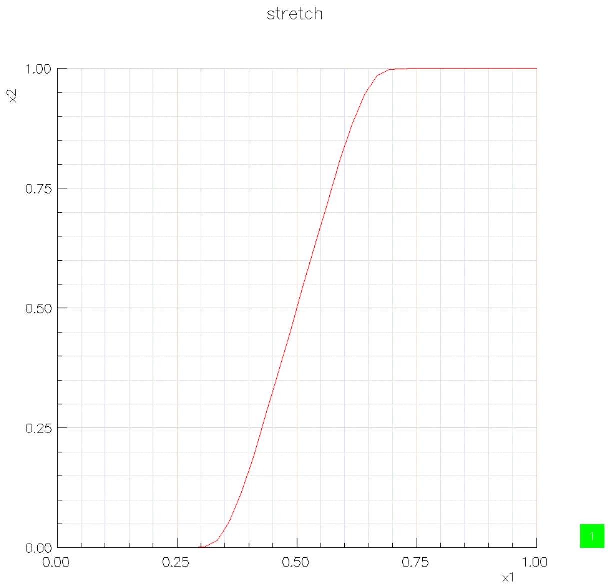
Stretching function: inverseHyperbolicTangent, 1 interval, $d_0 = 2.$, $e_0 = 10.$, $f_0 = .5$, $f_1 = 1.5$. This function will have grid spacing that is twice as small for $r > .5$



Stretching function: hyperbolicTangent, $a_0 = 0.$, $a_r = 1.$, $a_1 = -.9a_0/b_1$, $b_1 = 5.$, $c_1 = .5$. This function will concentrate grid points near $r = .5$



Stretching function: exponential, $a_0 = 0.$, $a_r = 1.$, $a_1 = 1.$, $b_1 = 5.$, $c_1 = .5$. This function will have grid spacing that concentrated near $r = 0$.



Stretching function: exponentialBlend. This function is exactly $0.$ for $r < \frac{1}{4}$ and exactly 1 for $r > \frac{3}{4}$.

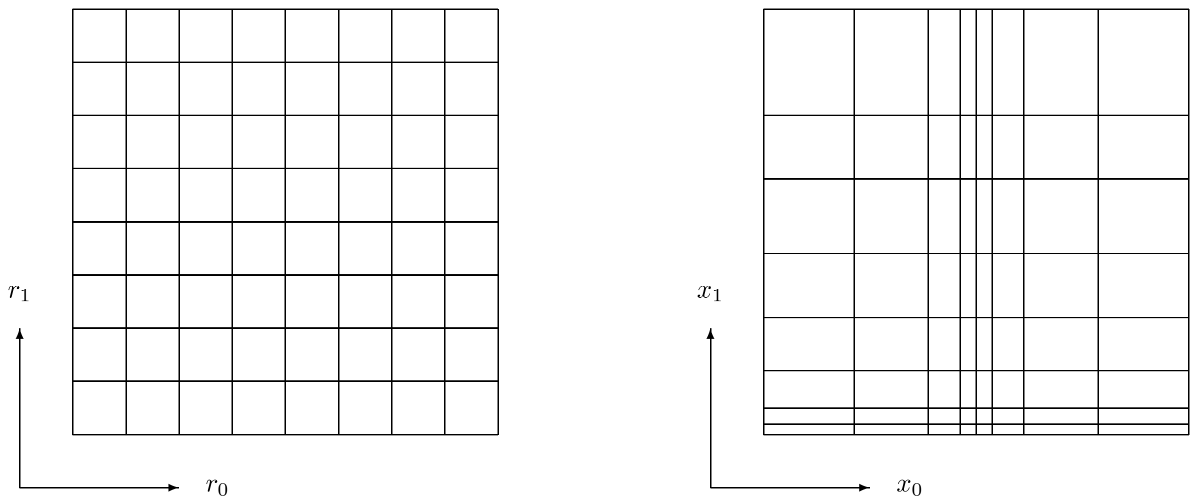


Figure 19: The StretchedSquare class can be used to stretch grids lines on the unit square or unit cube. The class defines a mapping from the parameter space (r_0, r_1) to the parameter space (x_0, x_1). Grids lines can be stretched on other Mapping's (such as an Annulus) by using the `StretchTransform` class which composes a Mapping with a `StretchedSquare`.

36 StretchedSquare: stretch grid lines on the unit interval

36.1 Description

This mapping uses the stretching functions of the `StretchMapping` class to stretch grid lines on the unit square or unit cube. The `StretchedSquare` defines a mapping from parameter space to parameter space. It can be used to stretch grids lines on other mappings (such as an annulus) using the `StretchTransform` class.

37 StretchTransform: stretch grid lines of an existing mapping

37.1 Description

This mapping can be used to reparameterize another mapping by stretcing the grid lines in the parameter directions. It does this by composing the `StretchedSquare` mapping with the given mapping. The `StretchedSquare` mapping in turn uses the `StretchMapping` to create stretching functions.

37.2 Constructors

`StretchTransform()`	Default constructor

37.3 Data Members

37.4 Member Functions

`void map( ... )`	evaluate the mapping and derivative
`void inverseMap( ... )`	evaluate the inverse mapping and derivative
`void get( const Dir & dir, const String & name)`	get from a database file
`void put( const Dir & dir, const String & name)`	put to a database file

38 Sweep Mapping

The SweepMapping can be used to create these types of mapping's,

sweep : sweep a curve or surface in the $x-y$ plane along a 3D curve.

extrude : extrude a curve or surface in the z direction.

tabulated-cylinder : generate a surface from a 3D curve by extruding along a specified line.

38.1 Sweep

The **sweep** option of the SweepMapping will take a planar reference surface $\mathbf{S}(r_1, r_2)$ (or reference curve $\mathbf{S}(r_1)$) and form a three-dimensional volume (or surface) by sweeping the reference surface along a 3D 'sweep-curve' $\mathbf{C}(r_3)$.

The formula defining the sweep mapping is

$$\mathbf{X}(r_1, r_2, r_3) = \Big\{ \mathbf{M}(\mathbf{r}) \ [\mathbf{S}(r_1, r_2) - \mathbf{c}_0] \Big\} \, \alpha(r_3) + \mathbf{C}(r_3)$$

where

$\mathbf{S}$	: the reference surface (or reference curve) to be swept.
$\mathbf{C}$	: the curve used for sweeping, the *sweep-curve*.
$\mathbf{M}$	: a rotation-translation matrix defined from $\mathbf{C}$ and $\mathbf{S}$.
$\alpha(r_3)$	: a scaling function.
$\mathbf{c}_0$	a vector used to centre the sweep mapping in different ways.

The vector $\mathbf{c}_0$ determines the centering of the SweepMapping with respect to the reference surface. There are three options for specifying the centering of the SweepMapping,

$\mathbf{c}_0 = 0$	: the centering is based on the sweep curve
$\mathbf{c}_0 = \overline{\mathbf{S}}(\cdot, \cdot)$	: the centering is based on the reference surface.
$\mathbf{c}_0 =$ user-specified	: the centering is user specified.

Here $\overline{\mathbf{S}}(\cdot, \cdot)$ is the centroid of the reference surface.

The initial rotation-translation matrix $\mathbf{M}(r_1, r_2, 0)$ will be chosen to translate the reference surface so that it's centroid, $\overline{\mathbf{S}}(\cdot, \cdot)$, is located at the point $\mathbf{c}_0$. The centroid is defined as the average value of the grid point locations,

$$\overline{\mathbf{S}}(\cdot, \cdot) = \frac{\sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \mathbf{S}_{i_1, i_2}}{(n_1 + 1)\,(n_2 + 1)} \, .$$

$\mathbf{M}(r_1, r_2, 0)$ will also rotate the reference surface to align with the tangent to the sweep-curve. After this rotation the the normal to the rotated reference surface will be parallel to the initial tangent of the the sweep-curve, $\mathbf{C}'(0)$. The **orientation** parameter ($+1$ or -1) will determine whether the normal to $\mathbf{S}$ is in the same or opposite direction to the tangent to the sweep curve. Thus if the sweep mapping appears 'inside-out' one should change the orientation. Instead of changing the orientation one could also reverse the parameterization of the sweep curve.

This needs to be finished

Here is the old documentation.

Purpose:

Given a planar surface (or curve) $\mathbf{S}(r_1, r_2)$ (or $\mathbf{S}(r_1)$), and a $3D$ curve $\mathbf{C}(r_3)$, we would like to generate a $3D$ volume or surface by sweepping $\mathbf{S}$ perpendicularly to $\mathbf{C}$ in such a way that the center of each $\mathbf{S}_k$ ring lie on the curve $\mathbf{C}$. At $r_3 = 0$, it is assumed that $\mathbf{S} = S_0$ is orthogonal to $\mathbf{C}$ and the tangent to $\mathbf{C}$ coincide with the normal $\mathbf{n}$ to $\mathbf{S}$. To make sure that the center of $\mathbf{S} = S_0$ lies at $\mathbf{C}(0)$, We first find the center (x_0, y_0, z_0) as the average of all the points that make up the sweep surface $\mathbf{S}$, namely

$$\mathbf{x}_0 = \frac{\sum_{i=0}^{n} \mathbf{x}_i}{n+1},$$

Then a translation that maps $\mathbf{C}(0)$ to (x_0, y_0, z_0) is applied to $\mathbf{C}$.

Strategy:
With a sufficient number of grid points in each direction, we incrementally compute the matrix transformation to be used the following way. At $k = 0$ corresponding to $r_3 = 0$, the identity matrix is used since $\mathbf{S}$ and $\mathbf{C}$ satisfy the required conditions and $\mathbf{S}_0 = \mathbf{S}$. For $k > 0$, the ring $\mathbf{S}_k$ is gotten from the ring $\mathbf{S}_{k-1}$ the following way:
A translation that maps the center of $\mathbf{S}_{k-1}$ (which is the same point as $\mathbf{C}_{k-1}$) to the point $\mathbf{C}_k$ is applied to $\mathbf{S}_{k-1}$. A rotation is then applied to the resulting points is such a way that the unit normal to the surface $\mathbf{S}_{k-1}$ coincides with the tangent to the curve $\mathbf{C}$ at the point C_k. To implement this, the unit vector $\mathbf{n}_0$ of the surface $\mathbf{S}_{k-1}$ is chosen to be the first vector in a new orthonormal basis. The second basis vector $\mathbf{n}_1$ is given by $\mathbf{n}_1 = \frac{n_0 \times t}{\|n_0 \times t\|}$ where $\mathbf{t} = \frac{\partial C(r_3 + \Delta r_3)}{\partial r_3}$. The third basis vector $\mathbf{n}_2$ is given by $\frac{\mathbf{n}_0 \times n_1}{\|n_0 \times n_1\|}$. In the new coordinate system, the rotation is about $\mathbf{n}_1$ with center at C_k. Since n_0 is rotated to coincide with t, the rotation angle is given by $\cos\theta = n_0 \cdot t$ and $\sin\theta = t \cdot n_2$. The overall matrix transformation is therefore a product of three matrices; first the matrix transformation from the canonic basis of the 3D vector space to the basis (n_0, n_1, n_2), the rotation of angle θ with center $(0, 0, 0)$ around $\mathbf{n}_1$ and finally the matrix transformation from the basis (n_0, n_1, n_2) to the canonic basis.
For the simplification of the mapping calculations, the discrete values of the global transformation $M(r_{1k}, r_{2k}, r_{3k})$ are considered as the points for three splines. With these splines we can calculate the image of any triplet (r_1, r_2, r_3). If $\alpha(r_3)$ is the value of the scalar we will multiply (also stored in a spline), the image $\mathbf{X}(r_1, r_2, r_3)$ is given by

$$\mathbf{X}(r_1, r_2, r_3) = \{M(r_1, r_2, r_3) * [\mathbf{S}(r_1, r_2) - \mathbf{C}(0)]\}\, \alpha(r_3) + \mathbf{C}(r_3)$$

Remark
At the limit $(\Delta r_3 \rightarrow 0)$ corresponding to the continuous case, the basis (n_0, n_1, n_2) becomes proportional to $\frac{\partial C(r_3)}{\partial r_3}, \frac{\partial^2 C(r_3)}{\partial r_3^2}, \frac{\partial C(r_3)}{\partial r_3} \times \frac{\partial^2 C(r_3)}{\partial r_3^2}$. In fact when Δr_3 is very small then

$$\begin{aligned}
n_1 &\approx \frac{\partial C(r_3)}{\partial r_3} \times \frac{\partial C(r_3 + \Delta r_3)}{\partial r_3} \\
n_1 &\approx \frac{\partial C(r_3)}{\partial r_3} \times \left(\frac{\partial C(r_3)}{\partial r_3} + \Delta r_3 \frac{\partial^2 C(r_3)}{\partial r_3^2} + \cdots \right) \\
&\approx \Delta r_3 \frac{\partial C(r_3)}{\partial r_3} \times \frac{\partial^2 C(r_3)}{\partial r_3^2}
\end{aligned}$$

Acknowledgement: Thanks to Thomas Rutaganira for creating the first version of the SweepMapping.
Here are the description of some functions of the class

38.2 Constructor

SweepMapping(Mapping *sweepmap = NULL,
Mapping *dirsweepmap = NULL,
Mapping *scale = NULL,
const int domainDimension0 =3)

Description: Define a sweep mapping or an extruded mapping.

Build a mapping defined by a sweep surface or curve (a mapping with domainDimension=2 rangeDimension=3 or domainDimension=1, rangeDimension=3) and a sweep curve or line (domainDimension=1, rangeDimension=3).

sweepmap (input) : is the mapping for the sweep surface or curve; default: an annulus with inner radius=0 and outer radius=1

dirsweepmap (input) : The mapping for the sweep curve; default: a half circle of radius=4.

scale (input) : to scale up (> 1) or down $(0 < s < 1)$; default 1.

Author: Thomas Rutaganira.

Changes: WDH + AP

38.3 SetSweepSurface

void
setSweepSurface(Mapping *sweepmap)

Description: Specify the mapping to use as the sweepMap, a 3D surface or a 3D curve. If it is a 3D surface, the resulting SweepMapping will be a 3D volume and if it is a 3D curve, the SweepMapping will be a 3D surface.

38.4 setCentering

int
setCentering(CenteringOptionsEnum centering)

Description: Specify the centering.

centering (input) : Specify the manner in which the reference surface should be centered. One of **useCenterOfSweepSurface**, **useCenterOfSweepCurve** or **specifiedCenter**. See the documentation for further details.

38.5 setOrientation

int
setOrientation(real orientation_ =1.)

Description: Specify the orientation of the sweepmapping, +1 or -1. When the sweep surface is rotated to align with the sweep curve it may face in a forward or reverse direction depending on the orientation. Thus if a swept surface appears 'inside-out' one should change the orientation.

38.6 setExtrudeBounds

int
setExtrudeBounds(real za_ =0.,
real zb_ =1.)

Description: Specify the bounds on an extruded mapping.

za_,zb_ (input) :

38.7 setStraightLine

int
setStraightLine(real lx =0. */, real ly /* =0. */, real lz /* =1.)

Description: Specify the straight line of a tabulated cylinder mapping

lx,ly,lz (input) :

38.8 SetSweepCurve

void
setSweepCurve(Mapping *dirsweepmap)

Description: Specify the mapping to use as the curve to sweep along (a 3D curve).

38.9 SetScaleSpline

void
setScale(Mapping *scale)

Description: Specify the mapping to use as the curve to sweep along (a 3D curve).

38.10 setMappingProperties

int
setMappingProperties()

Access: protected.

Description: Initialize the parameters of the sweep mapping.

38.11 FindRowSplines

void SweepMapping
findRowSplines(void)

Description: This function initializes the splines rowSpline0, 1, 2 that will gives the matrix transformation as well as its derivatives for the mapping calculations. A point of the spline gives a row for the matrix transformation.

38.12 map

void
map(const realArray & r, realArray & x, realArray & xr, MappingParameters & params)

Description: Use the transformations defined by rowSpline0, rowSpline1, and rowSpline2 and the additional scaling mapping to compute the image(s) and/or the derivatives for the parameter point(s) defined by r.

38.13 Examples

The command file **Overture/sampleMappings/aorticArch.cmd** generates the mappings for a model of the aortic arch shown in figure 20.

Figure 21 shows the grids generated by the SweepMapping for a model of a stadium.

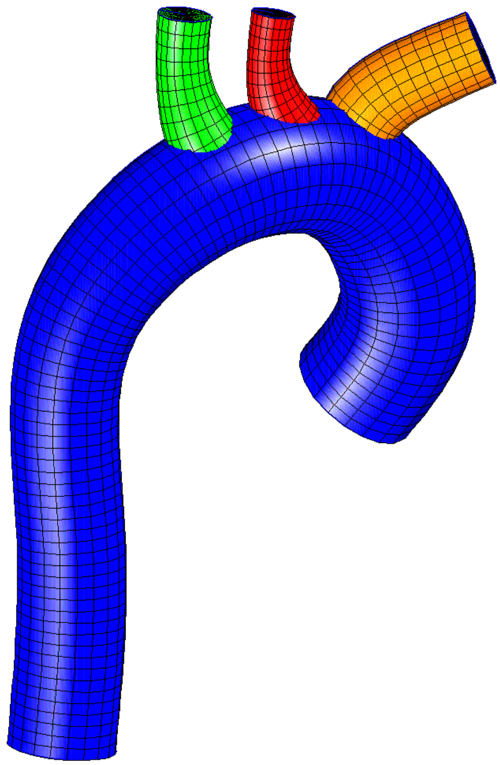


Figure 20: The SweepMapping is used to generate mappings for the aortic arch.

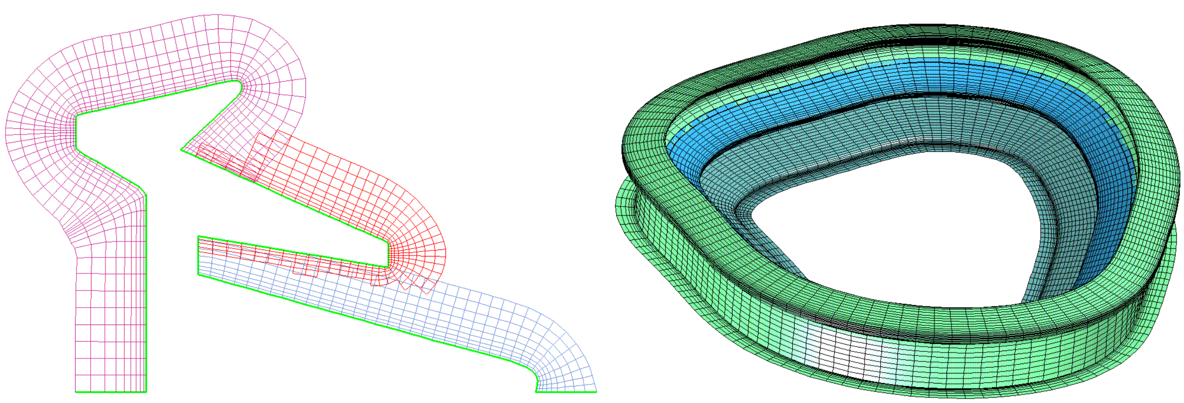


Figure 21: The SweepMapping is used to generate mappings for a stadium.

39 TFIMapping: Transfinite-Interpolation

Thanks to Brian Miller for help with this Mapping.

Use this Mapping to create a transfinite-interpolation mapping (also known as a Coon's patch). Transfinite-interpolation creates a patch (grid) in 2D (3D) using curves (surfaces) that define the boundaries. The quality of this algebaric grid is strongly dependent on the parameterizations of the boundary curves. // In its simplest form this mapping blends two given curves to create a grid in the region between the curves. Given the two curves $\mathbf{c}_i(r_1)$ the linear interpolation formula is

$$\mathbf{x}(r_1, r_2) = \mathbf{c}_0(r_1)(1 - r_2) + \mathbf{c}_1(r_1)r_2$$

With an hermite interpolation it is possible to also specify the r_2 derivative of the patch at the boundaries. The formula for hermite interpolation between two curves is

$$\mathbf{x}(r_1, r_2) = \mathbf{c}_0(r_1)\Phi_0(r_2) + \mathbf{c}(r_1)\Phi_1(r_2) \; + \; \dot{\mathbf{c}}_0(r_1)\Psi_0(r_2) + \dot{\mathbf{c}}(r_1)\Psi_1(r_2)$$

where the blending functions are given by

$$\begin{aligned} \Phi_0(r) &= (1 + 2r)(1 - r)^2 \qquad & \Phi_1(r) &= (3 - 2r)r^2 \\ \Psi_0(r) &= r(1 - r)^2 \qquad & \Psi_1(r) &= (r - 1)r^2 \end{aligned}$$

The patch will satisfy the conditions

$$\mathbf{x}(r_1, i) = \mathbf{c}_i(r_1) \;\text{ and }\; \frac{\partial}{\partial r_2}\mathbf{x}(r_1, 0) = \dot{\mathbf{c}}_i(r_1) \quad i = 0, 1$$

The grid lines will be normal to the boundary provided that we choose the derivatives $\dot{\mathbf{c}}_i$ to be proportional to the normal vectors. The scaling of this vector will determine the grid spacing near the boundary. We choose

$$\dot{\mathbf{c}}_i(r) = \|\mathbf{c}_1(r) - \mathbf{c}_0(r)\| \; \mathbf{n}_i(r)$$

where $\mathbf{n}_i(r)$ is the unit normal vector and $\|(a, b)\| = \sqrt{a^2 + b^2}$.

The derivative of the hermite-interpolant involves the second derivative of the boundary curves and is given by

$$\frac{\partial}{\partial r_1}\mathbf{x}(r_1, r_2) = \frac{\partial}{\partial r_1}\mathbf{c}_0(r_1)\Phi_0(r_2) + \frac{\partial}{\partial r_1}\mathbf{c}(r_1)\Phi_1(r_2) \; + \; \frac{\partial}{\partial r_1}\dot{\mathbf{c}}_0(r_1)\Psi_0(r_2) + \frac{\partial}{\partial r_1}\dot{\mathbf{c}}_1(r_1)\Psi_1(r_2)$$

where

$$\frac{\partial}{\partial r_1}\dot{\mathbf{c}}_i = \frac{(\mathbf{c}_1 - \mathbf{c}_0) \cdot (\mathbf{c}_{1,r_1} - \mathbf{c}_{0,r_1})\dot{\mathbf{c}}_i}{\|\mathbf{c}_1 - \mathbf{c}_0\|^2} - \frac{(\dot{\mathbf{c}}_i \cdot \mathbf{c}_{i,r_1 r_1})\mathbf{c}_{i,r_1}}{\|\mathbf{c}_{i,r_1}\|^2}$$

More generally a transfinite-interpolation mapping can interpolate 4 curves in 2D/3D or up to 6 curves in 3D. Define

$$\begin{aligned} \mathbf{c}_{\text{left}} &: \text{side corresponding to } r_1 = 0. \\ \mathbf{c}_{\text{right}} &: \text{side corresponding to } r_1 = 1. \\ \mathbf{c}_{\text{bottom}} &: \text{side corresponding to } r_2 = 0. \\ \mathbf{c}_{\text{top}} &: \text{side corresponding to } r_2 = 1. \\ \mathbf{c}_{\text{back}} &: \text{side corresponding to } r_3 = 0. \\ \mathbf{c}_{\text{Front}} &: \text{side corresponding to } r_3 = 1. \end{aligned}$$

The notation 'left', 'right', etc. comes from the fact that when a cube is plotted in the graphics window the left side is the face $r_1 = 0$, the right face is $r_1 = 1$ etc.

In two dimensions, when 4 sides are specified, the formula for linear interpolation is

$$\begin{aligned} \mathbf{c}(r_1, r_2) = {} & \mathbf{c}_{\text{left}}(r_2)(1 - r_1) + \mathbf{c}_{\text{right}}(r_2)r_1 \\ & + \mathbf{c}_{\text{bottom}}(r_1)(1 - r_2) + \mathbf{c}_{\text{top}}(r_1)r_2 \\ & - \Big\{(1 - r_2)((1 - r_1)\mathbf{c}_{\text{left}}(0) + r_1\mathbf{c}_{\text{right}}(1)) + r_2((1 - r_1)\mathbf{c}_{\text{bottom}}(0) + r_1\mathbf{c}_{\text{top}}(1))\Big\}. \end{aligned}$$

The last line in this formula represents a correction, a bilinear function that passes through the four corners, that ensures the mapping, $\mathbf{c}$, matches the four boundary curves. In three-dimensions with 6 sides specified

$$
\begin{aligned}
\mathbf{c}(r_1, r_2) = &\ \mathbf{c}_{\mathrm{left}}(r_2, r_3)(1 - r_1) + \mathbf{c}_{\mathrm{right}}(r_2, r_3) r_1 \\
&+ \mathbf{c}_{\mathrm{bottom}}(r_1, r_3)(1 - r_2) + \mathbf{c}_{\mathrm{top}}(r_1, r_3) r_2 \\
&+ \mathbf{c}_{\mathrm{back}}(r_1, r_2)(1 - r_3) + \mathbf{c}_{\mathrm{Front}}(r_1, r_2) r_3 \\
&- \Big\{(1 - r_2)((1 - r_1)\mathbf{c}_{\mathrm{left}}(0, r_3) + r_1 \mathbf{c}_{\mathrm{right}}(0, r_3)) + r_2((1 - r_1)\mathbf{c}_{\mathrm{bottom}}(0, r_3) + r_1 \mathbf{c}_{\mathrm{top}}(1, r_3)) \\
&+ (1 - r_2)((1 - r_3)\mathbf{c}_{\mathrm{bottom}}(r_1, 0) + r_3 \mathbf{c}_{\mathrm{top}}(r_1, 1)) + r_2((1 - r_3)\mathbf{c}_{\mathrm{back}}(r_1, 1) + r_3 \mathbf{c}_{\mathrm{Front}}(r_1, 1)) \\
&+ (1 - r_3)((1 - r_1)\mathbf{c}_{\mathrm{back}}(0, r_2) + r_1 \mathbf{c}_{\mathrm{Front}}(1, r_2)) + r_3((1 - r_1)\mathbf{c}_{\mathrm{left}}(r_2, 1) + r_1 \mathbf{c}_{\mathrm{right}}(r_2, 1))\Big\} \\
&+ \Big[(1 - r_3)[(1 - r_2)((1 - r_1)\mathbf{c}(0, 0, 0) + r_1 \mathbf{c}(1, 0, 0)) + r_2((1 - r_1)\mathbf{c}(0, 1, 0) + r_1 \mathbf{c}(1, 1, 0))) \\
&+ r_3[(1 - r_2)((1 - r_1)\mathbf{c}(0, 0, 1) + r_1 \mathbf{c}(1, 0, 1)) + r_2((1 - r_1)\mathbf{c}(0, 1, 1) + r_1 \mathbf{c}(1, 1, 1)))\Big].
\end{aligned}
$$

In the 3d case we must first subtract off corrections as in 2D (3 such corrections) and then add back a trilinear function that passes through the 8 vertices to ensure that all 6 sides are matched.

39.1 Compatibility conditions

The above TFI formulae will only give a reasonable grid if

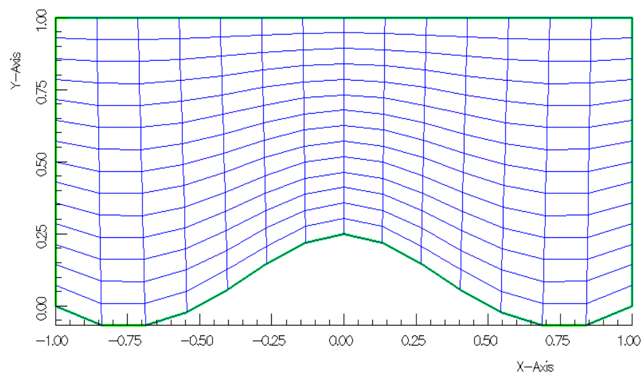
1. The curves that define the faces match at vertices (and edges in 3d).
2. The curves are parameterized in the ‘same direction’, otherwise the grids lines could cross.
3. Curves on opposite sides are parameterized in a similar way.

Even if all these conditions are met the grid lines may still cross if the boundary curves are strange enough.

39.2 Examples

39.2.1 2D linear TFI mapping with 2 sides specified

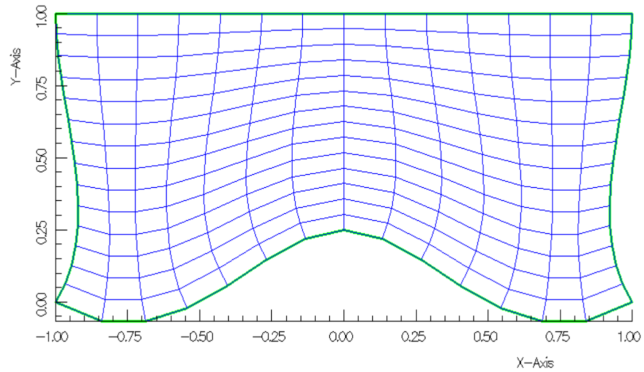
```
*
* Create a patch with transfinite interpo
*
* create a line for the top boundary
line
  number of dimensions
   2
  specify end points
    -1. 1. 1. 1.
exit
* create a spline for the bottom boundary
spline
  enter spline points
    5
    -1. 0.
    -.5 0.
    0. .25
    .5 .0
    1. 0.
exit
* create a tfi patch
tfi
  choose bottom curve
    spline
  choose top curve
    line
  mappingName
    tfi
  pause
 exit
 check
   tfi
```



A grid created with linear trans-finite interpolation

39.2.2 2D hermite TFI mapping with 2 sides specified

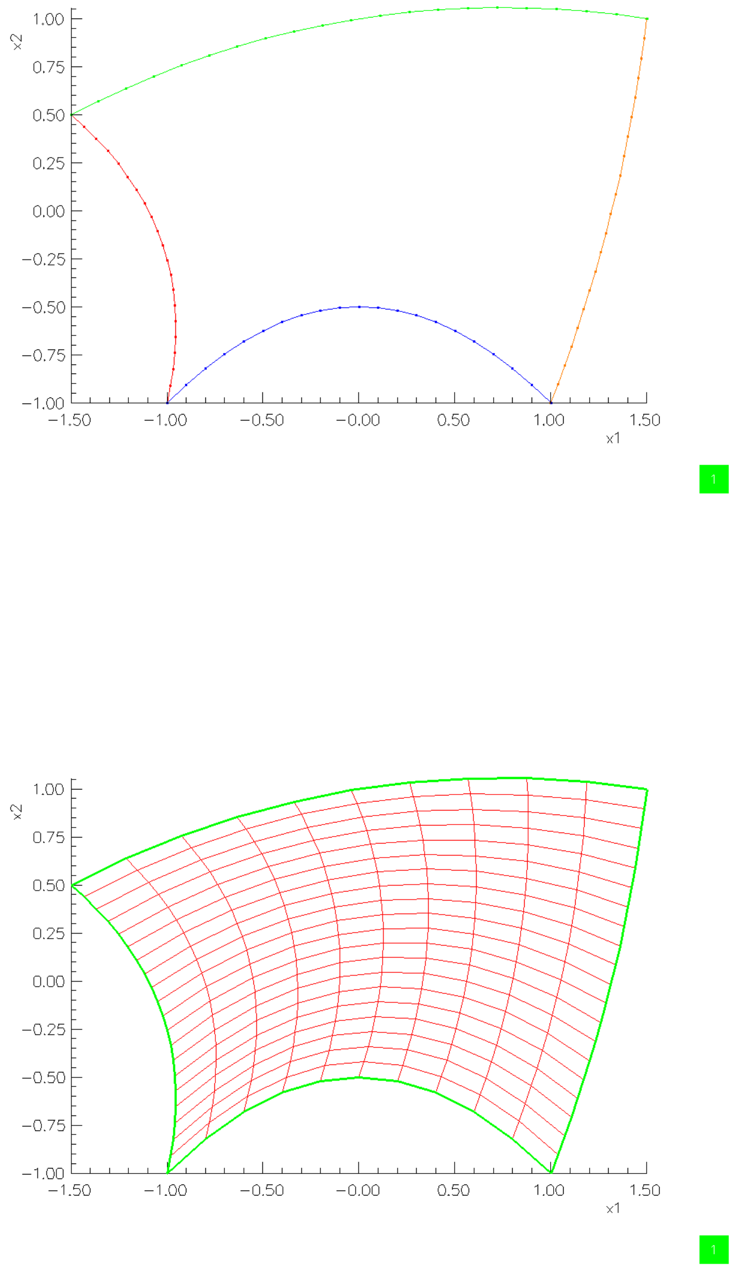
```
*
* Create a patch with transfinite interpo
*
* create a line for the top boundary
line
  number of dimensions
   2
  specify end points
    -1. 1. 1. 1.
exit
* create a spline for the bottom boundary
spline
  enter spline points
    5
    -1. 0.
    -.5 0.
    0. .25
    .5 .0
    1. 0.
exit
* create a tfi patch
tfi
  choose bottom
    spline
  choose top
    line
  hermite interpolation
  mappingName
    tfi
  pause
 exit
 check
   tfi
```



A grid created with hermite trans-finite interpolation; grid lines are normal to the bottom and top boundaries

39.2.3 2D linear TFI mapping with 4 sides specified

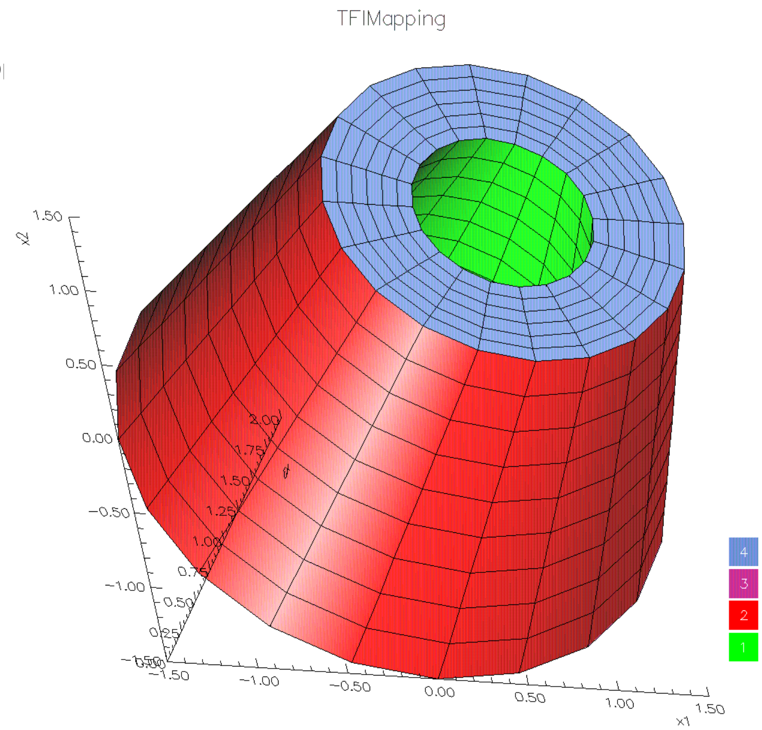
```
spline
  enter spline points
    3
    -1.0 -1.0
    0.0 -.5
    1.0 -1.0
  lines
    21
  mappingName
    bottomSpline
  exit
spline
  enter spline points
    3
    -1.5 .5
    0.0 1.0
    1.5 1.0
  lines
    21
  mappingName
    topSpline
  exit
spline
  enter spline points
    3
    -1.0 -1.0
    -1.0 -.25
    -1.5 .5
  lines
    21
  mappingName
    leftSpline
  exit
spline
  enter spline points
    3
    1.0 -1.0
    1.25 -.25
    1.5 1.0
  lines
    21
  mappingName
    rightSpline
  exit
tfi
  mappingName
    tfi2d4
  choose bottom
    bottomSpline
  choose top
    topSpline
  choose left
    leftSpline
  choose right
    rightSpline
  pause
  exit
check
  tfi2d4
```



A grid created with linear trans-finite interpolation, all four sides are specified. The top figure shows the 4 boundary curves before interpolation.

39.2.4 3D linear TFI mapping with 2 sides specified

```
*
*  3D TFI Mapping between two Annulus map
*
* make the annulus for the top
Annulus
  make 3d (toggle)
    2.
  mappingName
    top-annulus
  exit
* make the annulus for the bottom
Annulus
  outer radius
    1.5
  inner radius
    1.
  make 3d (toggle)
    0.
  mappingName
    bottom-annulus
exit
tfi
  choose back curve
    bottom-annulus
  choose front curve
    top-annulus
  boundary conditions
    -1 -1 1 2 3 4


```



A grid created with linear trans-finite interpolation between two Annulus mappings.

39.3 setSides

int
setSides(Mapping *left =NULL,
Mapping *right =NULL,
Mapping *bottom =NULL,
Mapping *top =NULL,
Mapping *front =NULL,
Mapping *back =NULL)

Purpose: Build a TFIMapping and supply curves that define the boundaries. Specify 0, 2, 4 or 6 curves. The Trans-Finite-Interpolation (TFI) Mapping (also known as a Coon's patch) will interpolate between the boundary curves to define a mapping in the space between. See the documentation for further details.

left, right (input): curves for $r_1 = 0$ and $r_1 = 1$.

bottom, top (input): curves for $r_2 = 0$ and $r_2 = 1$.

front, back (input): curves for $r_3 = 0$ and $r_3 = 1$ (3D only).

39.4 flipper

int
flipper()

Purpose: Try to flip the curve parameterizations to make the mapping non-singular.

Notes: Fix up a TFIMapping that turns inside out because the bounding curves are not parameterized in compatible ways.

39.5 map

void
map(const realArray & r, realArray & x, realArray & xr, MappingParameters & params)

Purpose: Evaluate the TFI and/or derivatives.

39.6 update

int
update(MappingInformation & mapInfo)

Purpose: Interactively create and/or change the TFI mapping.

mapInfo (input): Holds a graphics interface to use.

40 TrimmedMapping: define a trimmed surface in 3D

40.1 Description

A trimmed surface consists of a standard (i.e. logically rectangular) mapping (”surface”) which has regions removed from it. The portions removed are defined by curves in the parameter space of the mapping. The IN-ACTIVE part of the trimmed surface is any point that is outside the outer boundary or inside any of the inner curves. Thus one inner curve cannot be inside another inner curve. None of the trimming curves are allowed to intersect each other.

Here is how you should evaluate a trimmed mapping (accessing the mask array to indicate whether the point is inside or outside):

```
TrimmedMapping trim;
... assign the TrimmedMapping somehow ...
RealArray r(10,2), x(10,3), xr(10,3,2);
... assign r ....
MappingParameters params;   // we need to pass this option argument to "map"
trim.map(r,x,xr,params);
IntegerArray & mask = params.mask;  //  mask(i) = 0 if point is outside, =1 if inside
for( int i=0; i<9; i++; )
{
  if( mask(i)==0 )
    // point is outside, x(i,0:2) are the coordinates of the untrimmed surface at r(i,0:1)
  else
    // point is inside, x(i,0:2) are the coordinates of the trimmed surface at r(i,0:1)
}
```

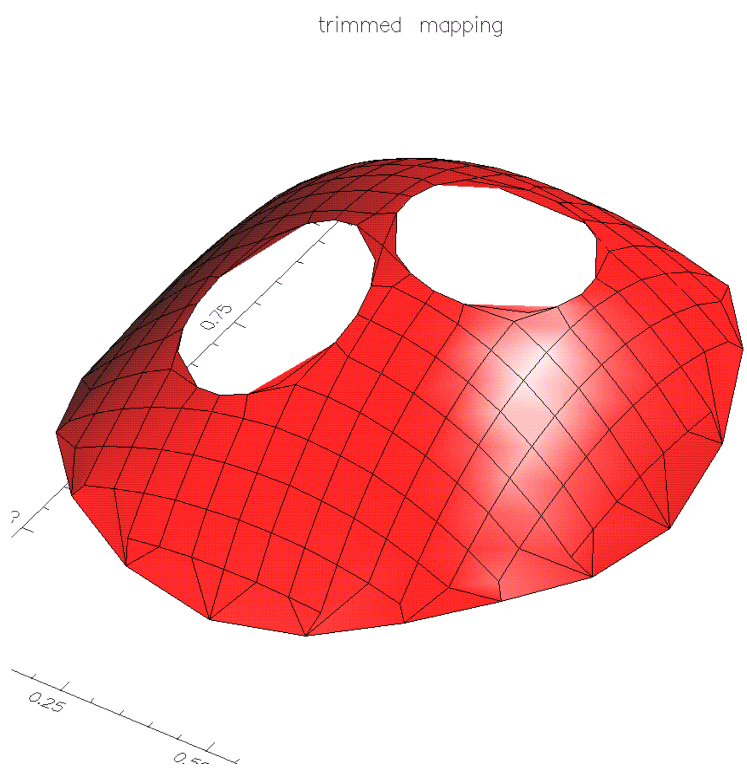


Figure 22: A trimmed mapping with an outer trimming curve and 2 inner trimming curves. To plot the mapping we project points which are just outside the trimmed region onto the boundary

40.2 Constructor

TrimmedMapping()

Purpose: Default Constructor

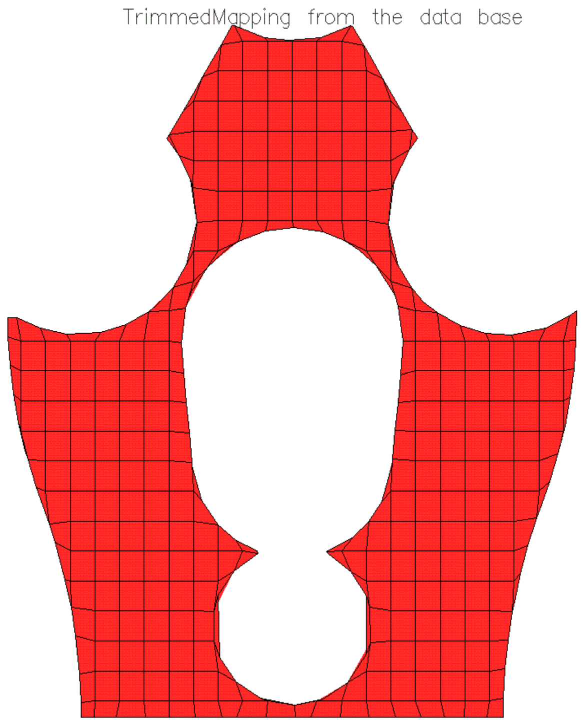


Figure 23: A trimmed mapping with an outer trimming curve and 1 inner trimming curve

40.3 Constructor

TrimmedMapping(Mapping & surface_,
Mapping *outerCurve_ =NULL,
const int & numberOfInnerCurves_ =0,
Mapping **innerCurve_ =NULL)

Purpose: Create a trimmed surface

surface (input) : surface to be trimmed

outerCurve (input) : curve defining the outer boundary, if NULL then the outer boundary is the boundary of surface

numberOfInnerCurves (input) : number of closed curves in the interior that trim the surface

General Notes: In order to evaluate a trimmed mapping we need to decide whether we are inside or outside. To make this determination faster, we divide the domain space (r) with a quadtree mesh: the domain is broken into 4 squares, each of which is subdivided into 4 again, recursively as needed. Each square is marked whether it lies inside the domain, outside, or partly in and partly out. It is quick to traverse the quadtree to find which square a given point is in. If the square is inside or outside we are done. If it is mixed, we usually have to check the point against only one curve to determine insideness.

40.4 Constructor

TrimmedMapping(Mapping & surface_,
const int & numberOfTrimCurves_ =0,
Mapping **trimCurves_ =NULL)

Purpose: Create a trimmed surface

surface (input) : surface to be trimmed

numberOfTrimCurves_ (input) : number of closed curves that trim the surface

trimCurves_ (input) : the trimming curves

General Notes: In order to evaluate a trimmed mapping we need to decide whether we are inside or outside. To make this determination faster, we divide the domain space (r) with a quadtree mesh: the domain is broken into 4 squares, each of which is subdivided into 4 again, recursively as needed. Each square is marked whether it lies inside the domain, outside, or partly in and partly out. It is quick to traverse the quadtree to find which square a given point is in. If the square is inside or outside we are done. If it is mixed, we usually have to check the point against only one curve to determine insideness.

40.5 setCurves

int
setCurves(Mapping & surface_,
const int & numberOfTrimCurves_ =0,
Mapping **trimCurves_ =NULL)

Purpose: Specify the surface and trimming curves.

surface (input) : surface to be trimmed

numberOfInnerCurves (input) : number of closed curves that trim the surface

trimCurves_ (input) : the oriented trim curves that trim the surface

40.6 setUnInitialized

void
setUnInitialized()

Description: Indicate that this Mapping is not up to date. This will destroy the triangulation used to plot it.

40.7 initializeTrimCurves

void
initializeTrimCurves()

Access: protected

Description: Compute trimming curve arclengths, areas, orientation, dRmin, dSmin.

40.8 addCurve

bool
addTrimCurve(Mapping *newCurve)

Purpose: Add a trimming curve to the surface

newCurve (input) : the new trim curve

returns : true if there were no problems with the trimming curve, false otherwise

40.9 deleteTrimCurve

bool
deleteTrimCurve(int curveToDelete)

Purpose: delete a specific trimming curve from the surface. Note : if the curve to be delete is the last counterclockwise curve, then a default trimming curve is built consisting of the untrimmed surface's boundary.

curveToDelete : index of the trim curve to be removed

returns : returns false if the last counterclockwise trim curve was removed resulting in the creation of a default outer curve.

40.10 deleteTrimCurve

bool
deleteTrimCurve(int numberOfCurvesToDelete, int *curvesToDelete)

Purpose: delete multiple trimming curves from the surface. Note : if the curve to be delete is the last counterclockwise curve, then a default trimming curve is built consisting of the untrimmed surface's boundary.

numberOfCurvesToDelete (input): the length of the array curvesToDelete

curvesToDelete (input): an array containing a list of the curves to be deleted

returns : returns false if the last counterclockwise trim curve was removed resulting in the creation of a default outer curve.

40.11 undoDelete

bool
undoLastDelete()

Purpose: undo the last call to deleteTrimCurve

returns : true if successfull, false otherwise

40.12 initializeQuadTree (protected)

void
initializeQuadTree(bool buildQuadTree =true)

Description: Initialize things needed by the quad-tree search and optionally build the quad-tree.

- initialize the bounding boxes for each of the trimming curves
- Make the array rCurve[c] point to the "grid" for each trimming curve
- determine the rBound array which holds the bounds on the unit square in which conatins the trimmed surface.

40.13 getOuterCurve

Mapping*
getOuterCurve()

Description: Return a pointer to the outer trimming curve.

40.14 getInnerCurve

Mapping*
getInnerCurve(const int & curveNumber)

Description: Return a pointer to the inner trimming curve number curveNumber.

curveNumber (input) : number of the trimming curve, between 0 and `getNumberOfInnerCurves()`. Return 0 if the curveNumber is invalid.

40.15 curveGoesThrough

bool
curveGoesThrough(const TMquad& square, const int& c, int& segstart, int& segstop) const

Access: public.

Description: Determine whether the polygonal curve c goes through the square quadtree node "square". If so, return true. One may specify starting and stopping segment numbers of the curve. These will be reset to indicate the curve segments which pass through the square. 0 and -1 mean to use all segments.

40.16 insideOrOutside

int
insideOrOutside(const realArray & rr, const int & c)

Access: protected.

Purpose: find if the point rr lies inside the curve c (actually inside the polygon defined by rCurve(0:n,0:1)) and return the distance to that curve.

Method: Use the routine from the mapping inverse to count how many times a vertical ray traced above the point crosses the polygon. NOTE: points exactly on the boundary are ”outside” by this definition

rr (input): point the the parameter space of the untrimmed surface.

c (input): curve number.

distance (output):

Return values: **+1** if the point is inside the outerCurve (c==1) or oustide the inner curve (c¿1)

-1 otherwise

40.17 insideOrOutside

int
insideOrOutside(const realArray & rr, const int & c, realArray & distance)

Access: protected.

Purpose: find if the point rr lies inside the curve c (actually inside the polygon defined by rCurve(0:n,0:1)) and return the distance to that curve. This routine calls the `insideOrOutside( const realArray & rr, const int & c )` function.

rr (input): point the the parameter space of the untrimmed surface.

c (input): curve number.

distance (output):

Return values: **+1** if the point is inside the outerCurve (c==1) or oustide the inner curve (c¿1)

-1 otherwise

40.18 findClosestCurve

int
findClosestCurve(const realArray & x,
intArray & cMin,
realArray & rC,
realArray & xC,
realArray & dist,
const int & approximate =TRUE)

/ N.B. I HAVE CHANGED THIS: Some changed in the specification are to match the actual code, and some changes in the code are to match the pre-existing specification. But the code has changed in that cMin=-2 has a new, special meaning. (jfp 0399)

Access: protected.

Description: Find the closest curve to a point and/or determine if the point is inside the curve.

x(R,.) (input) : points in the untrimmed surfaces parameter space

cMin(R) (input/output) : If cMin(base)¿0, then each cMin(i) is the number of the curve to be used for x(i,.). If cMin(base)¡0, then all curves will be checked, and on output cMin(i) will be the number of the curve nearest x(i,.) (This has been implemented only for the case where x is one point.) When cMin(base)¿0, cMin(i)==-2 means to skip computing the projection of x(i,). When cMin(base)=-2, nothing is computed.

xC(R,.) (output) : closest point on closest curve

dist (output) : dist(R) = minimum distance

approximate (input) : if TRUE only determine an approximation to the closest closest point on the closest curve (based on the nearest grid point on the polygonal representation of the curve).

40.19 findDistanceToACurve

int
findDistanceToACurve(const realArray & x,
IntegerArray & cMin,
realArray & dist,
const real & delta)

Access: protected.

Description: Find the approximate distance to a curve. (approximate if the distance ¿ deltaX)

x(R,.) (input) : points

cMin(R) (input/output) : if ¿0 on input then use this curve, on output it is the number of closest curve

dist (output) : dist(R) = approximate distance

40.20 map

void
map(const realArray & r, realArray & x, realArray & xr, MappingParameters & params)

Purpose: Evaluate the Trimmed and/or derivatives.

NOTE: In order to evaluate a trimmed surface you MUST provided a MappingParameters argument. Otherwise only the untrimmed mapping will be defined.

Notes: (1 The array params.mask(I) is returned with the values -1=outside, 0=inside

(2) if point i is outside the grid but near the trimmed boundary the array distanceToBoundary(i) is set to be the distance (in parameter space) of the point r(i,.) to the nearest trimming curve. The the point is far from the boundary, distance(i) is set to a large value.

40.21 map

void
mapGrid(const realArray & r,
realArray & x,
realArray & xr,
MappingParameters & params =Overture::nullMappingParameters())

Purpose: Map grid points and project grid points that cross a trimming curve onto the trimming curve. This routine is called by the plotting routine so that trimmed curves are properly plotted.

40.22 update

int
update(MappingInformation & mapInfo)

Purpose: Interactively create and/or change the Trimmed mapping.

mapInfo (input): Holds a graphics interface to use.

40.23 reportTrimCurveInfo

aString
reportTrimCurveInfo(Mapping *c, bool & curveok)

Purpose: return a string describing the state of a trim curve

c (input) : the curve in question

40.24 reportTrimmingInfo

aString
reportTrimmingInfo()

Purpose: return a string describing the state of the trimming

40.25 editTrimCurve

int
editTrimCurve(Mapping &trimCurve, MappingInformation & mapInfo)

Purpose: Interactively edit a trim curve

mapInfo (input): Holds a graphics interface to use.

40.26 editNurbsTrimCurve

int
editNurbsTrimCurve(NurbsMapping &trimCurve, MappingInformation & mapInfo)

Purpose: Interactively edit a nurbs trim curve

mapInfo (input): Holds a graphics interface to use.

41 UnstructuredMapping

The `UnstructuredMapping` class encapsulates the connectivity for an unstructured mesh. Currently the class supports the "Finite Element Zoo" collection of element types. This zoo consists of quadrilaterals and triangles in surface meshes and hexahedra, triangle prisms, pyramids and tetrahedra in volume meshes. A limited set of iterations through the mesh is now available and described below.

41.1 Implementation Details

Internally, the connectivity consists of two components : element internal connectivity provided by templates, and inter-element connectivity provided by linked lists. Since the supported element types are limited to the zoo, a canonical ordering of the vertices, faces, edges, sides, etc., in each element type can be constructed. Small template arrays and helper functions are used to map element-local indices of these components to global indices. Currently implemented three dimensional orderings are illustrated by Figure 24. In two dimensions the vertices and faces are simply ordered counter-clockwise starting from 0.

41.2 Iterations Through the Unstructured Connectivity

Iterating through the connectivity consists of using a few inlinable functions which abstract away the underlying representation, including the canonical orderings of the elements. Normally, a user will never even know the orderings exist (at least they should beware of depending upon them!). Iterations are also independent of the dimension of the mesh since the connectivity for all dimensions share the same nomenclature (eg. an edge in 2D is the same as an edge in 3D). The following subsections provide examples of how to navigate and use the limited set of iterations available in an `UnstructuredMapping`.

41.2.1 Element iteration

```
// assuming an UnstructuredMapping named um exists...
const IntegerArray &elements = um.getElements();
for ( int e=0; e<um.getNumberOfElements(); e++ ) {
    // ... do stuff with the element index
    elementScalar(e) = whatever;
}
```

41.2.2 Vertex iteration

```
// assuming an UnstructuredMapping named um exists...
const IntegerArray &vertices = um.getVertices();
for ( int v=0; v<um.getNumberOfVertices(); v++ ) {
    // ... do stuff with the vertex index
    vertexScalar(v) = whatever;
}
```

41.2.3 Iteration through the vertices in an element

```
for ( int e=0; e<um.getNumberOfElements(); e++ ) {
   for ( int v=0; v<um.getNumberOfVerticesThisElement(e); v++ ) {
      vGlobalIndex = um.elementGlobalVertex(e,v);
      // ... do stuff with global vertex index
      vertexScalar(vGlobalIndex) = whatever;
   }
}
```

41.2.4 Iteration through the faces

```
const IntegerArray &elements = um.getElements();
const IntegerArray &faceElements = um.getFaceElements();
for ( int f=0; f<um.getNumberOfFaces(); f++ ) {
   // get the elements on either side of the face
   int element0 = faceElements(f, 0);
   int element1 = faceElemetns(f, 1);
   // ... do stuff with the face and element indices
   faceScalar(f) = whatever;
   elementScalar(element0) += -face;
   elementScalar(element1) +=  face;
}
```

41.2.5 Iteration through the vertices in a face

```
const IntegerArray &elements = um.getElements();
for ( int f=0; f<um.getNumberOfFaces(); f++ ) {
   const IntegerArray &faceVertices = um.getFaceVertices(f);
   for ( int v=0; v<um.getNumberOfVerticesThisFace(f); v++ )
      vGlobalIndex = um.faceGlobalVertex(f,v);
      // ... do stuff with global vertex index
      vertexScalar(vGlobalIndex) = whatever;
   }
```

41.3 Enum Types

Currently `ElementType` is the only enum in `UnstructuredMapping`. `ElementType` enumerates the supported unstructured element types.

```
enum ElementType
  {
    triangle,
    quadrilateral,
    tetrahedron,
    pyramid,
    triPrism,
    septahedron,  // pray we never need...
    hexahedron,
    other,
    boundary
  };
```

In 2D, only triangles and quadrilaterals are supported. More types are supported in 3D, but in general these consist of the "finite element zoo". These elements are hexahedra or degenerate hexahedra. Currently the septahedron is not supported as this shape is rather unusual and rarely (?) encountered (7 nodes, 6 faces). `other` implies any shape not described by the previous enums and at this point could include arbitrary polyhedra (although the connectivity to support arbitrary polyhedra is not implemented). `boundary` elements are typically placeholders. `boundaries` will not have a specific geometry associated with them and may only consist of a limited set of connectivity.

41.4 File Formats

`UnstructuredMapping`'s can be written to two different kinds of files using the member functions `get` and `put`. Using `get` or `put` with an Overture `GenericDatabase` class as the first argument performs io directly to an Overture database file. During a `put`, the instance's arrays `node` and `element` are written to the database file. A `get` retrieves these arrays and reconstructs the connectivity. `UnstructuredMapping`'s can also be read/written to ASCII files using a simplified version of a format commonly called "ingrid", or "DYNA". This io method can be invoked by calling `get` and `put` with a string, the filename, as the first argument. The resulting file looks like :

```
A text header line (optional)
number of meshes, number of nodes, number of elements, domain dimension(optional), range dimension(optional)
node0 ID, x0, y0, z0
node1 ID, x1, y1, z1
.
.
.
nodeN ID, xN, yN,zN
element0 ID, tag0, n1,n2,n3,n4,n5,n6,n7,n8
element1 ID, tagN, node ID list
.
.
.
elementN ID, tagN, node ID list
```

Typically, Overture writes "OVERTUREUMapping" in the comment line and uses the optional spaces for the domain dimension and range dimension. These details, however, are not required and meshes from a variety of mesh generation tools have been read in. The node ID lists in the element lines are lists of global node ID's (listed in the node section), that are in each element. The ordering of the nodes in the list follows the canonical ordering described in Figure 24. Currently the code requires that the nodes be listed in ascending order of thier node ID's and that the node IDs be contiguous. By the way, the number of meshes in the file is ignored, only one mesh per file is supported at the moment.

41.5 Relationship to Normal Overture Mappings

While `UnstructuredMapping` inherits from class `Mapping`, it should be noted that there are a few caveats. By its very nature, the inverse does not exist for an `UnstructuredMapping`. Any use of an `UnstructuredMapping` in the context of mapping inverses should be prevented; all member functions dealing with inverses now throw exceptions. However, a domainDimension and rangeDimension are both still used in the mapping to help donote the difference between 2D meshes, 3D surface meshes and 3D volume meshes. With these exceptions, `UnstructuredMapping`s should play nicely with conventional sturctured ones. In particular, a structured mapping can be converted into an unstructured one by using the member function `buildFromAMapping`.

41.6 Member Function Descriptions

41.7 Constructor

UnstructuredMapping(int domainDimension_ =3,
int rangeDimension_ =3,
mappingSpace domainSpace_ =parameterSpace,
mappingSpace rangeSpace_ =cartesianSpace)

Description: Default Constructor

41.8 Constructor

UnstructuredMapping()

Description: Default Constructor

41.9 addGhostElements

void
addGhostElements(bool trueOrFalse)

Description: Specify whether to add ghost elements to the unstructured mapping.

trueOrFalse (input): If true add ghost elements to the unstructured mapping.

41.10 getBoundaryFace

const intArray &
getBoundaryFace() const

Description: Return a list of boundary faces,

boundaryFace (return value) : faces on the boundary.

41.11 getGhostElements

const intArray &
getGhostElements() const

Description: Return a list of ghost elements.

boundaryFace (return value) : faces on the boundary.

41.12 getMask

const intArray &
getMask(EntityTypeEnum entityType) const

Description: Return a list of ghost elements.

boundaryFace (return value) : faces on the boundary.

41.13 getBoundaryFaceTags

const intArray &
getBoundaryFaceTags() const

Description: Return a list of the tags on each boundary face, usefull for boundary conditions

boundaryFaceTags (return value) : tags for faces on the boundary.

41.14 getNumberOfNodes

int
getNumberOfNodes() const

Description: Return the number of nodes.

41.15 getMaxNumberOfNodesPerElement

int
getMaxNumberOfNodesPerElement() const

Description: Return the maximum number of nodes per element (max over all elements).

41.16 getMaxNumberOfNodesPerElement

int
getMaxNumberOfFacesPerElement() const

Description: Return the maximum number of faces per element (max over all elements).

41.17 getMaxNumberOfNodesPerFace

int
getMaxNumberOfNodesPerFace() const

Description: Return the maximum number of nodes per face (max over all faces).

41.18 getNumberOfElements

int
getNumberOfElements() const

Description: Return the number of elements (such as the number of triangles on a 2d grid or 3d surface or the number of tetrahedra in a 3d grid).

41.19 getNumberOfFaces

int
getNumberOfFaces() const

Description: Return the number of faces.

41.20 getNumberOfBoundaryFaces

int
getNumberOfBoundaryFaces() const

Description: Return the number of faces.

41.21 getNumberOfEdges

int
getNumberOfEdges() const

Description: Return the number of edges.

41.22 getNodes

const realArray &
getNodes() const

Description: Return the list of nodes.

node (return value) : list of nodes, node(i,0:r-1) : (x,y) or (x,y,z) coordinates for each node, i=0,1,... r=rangeDimension

41.23 getElements

const intArray &
getElements() const

Description: Return the node information for each element.

element (return value) : defines the nodes that make up each element (e.g. triangle), element(i,n) index into the nodes array for the node n of element i, for now n=0,1,2 for triangles. Thus element i will have nodes (element(i,0),element(i,1),...)

41.24 getFaces

const intArray &
getFaces() const

Description: Return the connectivity information for each face.

face (return value) : defines the nodes that make up each face (e.g. triangle), face(i,n) index into the nodes array for the node n of face i,

41.25 getFaces

const intArray &
getFaceElements() const

Description: Return the connectivity information containing the elements adjacent to each face.

faceElements (return value) : defines the elements adjacent to each face, faceElements(i,e) index into the elements array for the element e of face i, for now e=0,1 since each face has two elements. For now, faces on a boundary return -1 for e=1.

41.26 getEdges

const intArray &
getEdges() const

Description: Return the connectivity information for each edge.

edge (return value) : defines the 2 nodes that make up each edge. face(i,n) index into the nodes array for the node n of face i, for now n=0,1. Thus edge 0 will have end points with node numbers (edge(i,0),edge(i,1))

41.27 getElementFaces

const intArray &
getElementFaces()

Description: Return the connectivety array describing the faces that belong to an element.

elementFaces (return value) : defines the faces that belong to an element. face=elementFaces(e,i) is the face for i=0,1,..

41.28 getTags

const intArray &
getTags() const

Description: Return the element tagging information.

tags (return value) : an integer tag for each element, defaults to 0 for every element

41.29 setElementDensityTolerance

void
setElementDensityTolerance(real tol)

Description: Specify the tolerance for determining the triangle density when building from a mapping. The smaller the tolerance the more triangles. Choose a value of zero to use the default number of elements

tol (input) : new tolerance.

41.30 setTags

void
setTags(const intArray &new_tags)

Description: Set the list of tags for each element;

tags (input) : an array the length of the number of elements containing an integer tag for each element (eg like material region identifier)

41.31 setNodesAndConnectivity

```
int
setNodesAndConnectivity( const realArray & nodes,
                         const intArray & elements,
                         int domainDimension_ =-1,
                         bool buildConnectivity =true)
```

Description: Supply a list of nodes and a list of connectivity information.

nodes (input) : nodes(i,0:r-1) (x,y) or (x,y,z) coordinates for each node, i=0,1,... r=rangeDimension

elements (input) : defines the nodes that make up each element (e.g. triangle), elements(i,n) index into the nodes array for the node n of element i, for now n=0,1,2 for triangles. Thus element 0 will have nodes (elements(i,0),elements(i,1),...) A value of elements(i,n)==-1 means no node is used. This option is used to specify elements with different numbers of nodes per elements. For example if one has quadrilaterals and triangles then set element(i,3)=-1 for triangles.

41.32 setNodesElementsAndNeighbours

```
int
setNodesElementsAndNeighbours(const realArray & nodes,
                              const intArray & elements,
                              const intArray & neighbours,
                              int numberOfFaces_ =-1,
                              int numberOfBoundaryFaces_ =-1,
                              int domainDimension_ =-1)
```

Description: Supply a list of nodes, elements and element neighbours. The element neighbours are used in building the connectivity information. This should be faster than using setNodesAndConnectivity.

nodes (input) : nodes(i,0:r-1) (x,y) or (x,y,z) coordinates for each node, i=0,1,... r=rangeDimension

elements (input) : defines the nodes that make up each element (e.g. triangle), elements(i,n) index into the nodes array for the node n of element i, for now n=0,1,2 for triangles. Thus element 0 will have nodes (elements(i,0),elements(i,1),...) A value of elements(i,n)==-1 means no node is used. This option is used to specify elements with different numbers of nodes per elements. For example if one has quadrilaterals and triangles then set element(i,3)=-1 for triangles.

neighbours (input) : a list of neighbours for each element. / numberOfFaces_ (input) : optionally supply the number of faces, if known. / numberOfBoundaryFaces_ (input) : optionally supply the number of boundary faces, if known.

41.33 setNodesAndConnectivity

int
setNodesAndConnectivity(const realArray & nodes,
const intArray & elements,
const intArray & faces,
const intArray & faceElements_,
const intArray & elementFaces_,
int numberOfFaces_ =-1,
int numberOfBoundaryFaces_ =-1,
int domainDimension_ =-1)

Description: Supply a list of nodes, elements and element neighbours. The element neighbours are used in building the connectivity information. This should be faster than using setNodesAndConnectivity.

nodes (input) : nodes(i,0:r-1) (x,y) or (x,y,z) coordinates for each node, i=0,1,... r=rangeDimension

elements (input) : defines the nodes that make up each element (e.g. triangle), elements(i,n) index into the nodes array for the node n of element i, for now n=0,1,2 for triangles. Thus element 0 will have nodes (elements(i,0),elements(i,1),...) A value of elements(i,n)==-1 means no node is used. This option is used to specify elements with different numbers of nodes per elements. For example if one has quadrilaterals and triangles then set element(i,3)=-1 for triangles.

faces (input):

faceElements_ (input):

elementFaces_ (input): / numberOfFaces_ (input) : optionally supply the number of faces, if known. / numberOfBoundary-Faces_ (input) : optionally supply the number of boundary faces, if known.

41.34 buildFromAMapping

intArray
buildFromAMapping(Mapping & map, intArray &maskin = nullIntArray())

Description: Builds an unstructured mapping from another mapping. There are no duplicate nodes. Degenerate elements occurring from coordinate singularities and periodic boundaries are detected and the appropriate element (hex, prism, pyramid, tet) is created in the UnstructuredMapping. For example, a spherical polar mesh will, in general, have all four element types with pyramids at the spherical singularity, tetrahedron connecting the pyramids to the polar axes, prisms along each polar axis and hexahedra everywhere else. A mask array can optionally be provided to exclude vertices/elements from the new UnstructuredMapping. However, building a new UnstructuredMapping from a masked UnstructuredMapping is NOT yet supported. The implementor is a bit lazy.

map (input) : Mapping to use.

maskin (input) : pointer to a vertex mask array to determine which nodes/elements to use

Returns : An IntegerArray mapping the vertices in the original Mapping to the vertices in the new UnstructuredMapping. If the value of the returned array is -1 at any vertex, then that vertex was masked out of the original mapping.

Comments : Currently the code implements a rather complex algorithm to assign vertex id's to the boundary nodes. The complexity of the coding is due to the possibility of polar singularities (with the possible occurance of a spherical singularity) as well as periodic boundaries. These special cases can occur on any side of any coordinate axis in 2 and 3d. The approach became more complicated than originally intended, there may be a more straightforward way and any suggestions are welcome.

41.35 printConnectivity

int
printConnectivity(FILE *file stdout)

Description:

41.36 printConnectivity

int
**checkConnectivity(bool printResults =true,
IntegerArray *pBadElements =NULL)**

Description: Perform consistency checks on the connectivity.

printResults (input): output the results if true.

pBadElements (input/output) : If not null, return a list of the bad Elements.

return value: number of errors found.

41.37 printStatistics

int
printStatistics(FILE *file =stdout)

Description: print some timing statistics. ==

Description: Build an unstructured grid using a triangulation algorithm. use this routine if the Mapping boundaries are poorly behaved so that the grid cells give poor quality triangles. == // In order to use the 2D triangulation function we convert the 3D grid points // x(r0,r1) into 2D arclength coordinates s(r0,r1) // ::display(x,"x"); // compute arclength positions (s0,s1) of each grid point. // ::display(s,"s"); // choose the max area for a triangle from the average area of a cell. // Choose nodes and faces from the boundary points of the arclength array // First make a list of faces and vertices on the boundaries. faces(numberOfFaces-1,1)=0; // periodic I1=Range(0,nx-2); // leave off the last point xyz(ia+i,0,R2)=s(nx-1-i,ny-1,R2); // reverse order xyz2(0,ia+i,R2)=s(0,ny-1-i,R2); // reverse order // ::display(faces,"faces"); // ::display(xyz,"xyz"); // Note that there may be new nodes introduced. // Make a DataPointMapping of the arclenght positions dpm.inverseMap(sPoints,r); // compute unit square coordinates for the arclength positions. map.map(r,nodes); // compute 3d positions of triangle nodes. ===

Description: Optimised version to build an unstructured mapping from another mapping. The connectivity information will also be built directly.

elementTypePreferred (input): Prefer these type of elements

For triangles the connectivity will usually look like:

```
    12    13   14     15
    X-----X-----X-----X
    |13 / |15 / |17 / |
    | /12 | / 14| /16 |
   8X-----X-----X-----X11
    | 7 / | 9 / | 11/ |
    | / 6 | / 8 | /10 |
   4X-----X-----X-----X7
    | 1 / | 3 / | 5 / |
    | / 0 | / 2 | / 4 |
    X-----X-----X-----X
```

```
        0     1     2     3
```

For quadrilaterals the connectivity will usually look like:

```
        12    13   14     15
        X-----X-----X-----X
        |     |     |     |
        | 6   | 7   | 8   |
       8X-----X-----X-----X11
        |     |     |     |
        | 3   | 4   | 5   |
       4X-----X-----X-----X7
        |     |     |     |
        | 0   | 1   | 2   |
        X-----X-----X-----X
        0     1     2     3
```

41.38 get from an ascii file

int
get(const aString & fileName)

Description: Read the unstructured grid from an ascii file.

fileName (input) : name of the file to save the results in.

41.39 put to an ascii file

int
put(const aString & fileName) const

Description: Save the unstructured grid to an ascii file.

fileName (input) : name of the file to save the results in.

41.40 findBoundaryCurves

int
findBoundaryCurves(int & numberOfBoundaryCurves, Mapping **& boundaryCurves)

Description: Locate boundary curves on a 3D surface – booth curve segments on the boundary.

numberOfBoundaryCurves (output) : number of boundary curves found.

boundaryCurves (output) : Boundary curves as spline mappings. **NOTE:** This routine will increment the reference count for you.

41.41 Constructor

// void

//==

/ /Description: build an unstructured mapping from a composite grid

/ /cg (input) : a composite grid that may or may not be a hybrid grid

/ /Comments : The composite grid has no restrictions, it could be an overlapping // grid or hybrid mesh. In the case of an overlapping grid, the UnstructuredMapping // essentially consists of overlapping sections and holes that have no connectivity // information. A hybrid mesh becomes one consistent UnstructuredMapping.

41.42 Constructor

UnstructuredMapping(int domainDimension_ =3,
int rangeDimension_ =3,
mappingSpace domainSpace_ =parameterSpace,
mappingSpace rangeSpace_ =cartesianSpace)

Description: Default Constructor

41.43 Constructor

UnstructuredMapping()

Description: Default Constructor

41.44 getNumberOfNodes

int
getNumberOfNodes() const

Description: Return the number of nodes.

41.45 getMaxNumberOfNodesPerElement

int
getMaxNumberOfNodesPerElement() const

Description: Return the maximum number of nodes per element (max over all elements).

41.46 getMaxNumberOfNodesPerElement

int
getMaxNumberOfFacesPerElement() const

Description: Return the maximum number of faces per element (max over all elements).

41.47 getMaxNumberOfNodesPerFace

int
getMaxNumberOfNodesPerFace() const

Description: Return the maximum number of nodes per face (max over all faces).

41.48 getNumberOfElements

int
getNumberOfElements() const

Description: Return the number of elements (such as the number of triangles on a 2d grid or 3d surface or the number of tetrahedra in a 3d grid).

41.49 getNumberOfFaces

int
getNumberOfFaces() const

Description: Return the number of faces.

41.50 getNumberOfBoundaryFaces

int
getNumberOfBoundaryFaces() const

Description: Return the number of faces.

41.51 getNumberOfEdges

int
getNumberOfEdges() const

Description: Return the number of edges.

41.52 getNodes

const realArray &
getNodes() const

Description: Return the list of nodes.

node (return value) : list of nodes, node(i,0:r-1) : (x,y) or (x,y,z) coordinates for each node, i=0,1,... r=rangeDimension

41.53 getElements

const intArray &
getElements() const

Description: Return the node information for each element.

element (return value) : defines the nodes that make up each element (e.g. triangle), element(i,n) index into the nodes array for the node n of element i, for now n=0,1,2 for triangles. Thus element i will have nodes (element(i,0),element(i,1),...)

41.54 getFaces

const intArray &
getFaces() const

Description: Return the connectivity information for each face.

face (return value) : defines the nodes that make up each face (e.g. triangle), face(i,n) index into the nodes array for the node n of face i, for now n=0,1,2 for triangles. Thus face 0 will have nodes (face(i,0),face(i,1),...)

41.55 getFaces

const intArray &
getFaceElements() const

Description: Return the connectivity information containing the elements adjacent to each face.

faceElements (return value) : defines the elements adjacent to each face, faceElements(i,e) index into the elements array for the element e of face i, for now e=0,1 since each face has two elements. For now, faces on a boundary return -1 for e=1.

41.56 getEdges

const intArray &
getEdges() const

Description: Return the connectivity information for each edge.

edge (return value) : defines the 2 nodes that make up each edge. face(i,n) index into the nodes array for the node n of face i, for now n=0,1. Thus edge 0 will have end points with node numbers (edge(i,0),edge(i,1))

41.57 getTags

const IntegerArray &
getTags() const

Description: Return the element tagging information.

tags (return value) : an integer tag for each element, defaults to 0 for every element

41.58 setTags

void
setTags(const IntegerArray &new_tags)

Description: Set the list of tags for each element;

tags (input) : an array the length of the number of elements containing an integer tag for each element (eg like material region identifier)

41.59 setNodesAndConnectivity

```
int
setNodesAndConnectivity( const realArray & nodes,
                         const intArray & elements,
                         int domainDimension_ =-1)
```

Description: Supply a list of nodes and a list of connectivity information.

nodes (input) : nodes(i,0:r-1) (x,y) or (x,y,z) coordinates for each node, i=0,1,... r=rangeDimension

elements (input) : defines the nodes that make up each element (e.g. triangle), elements(i,n) index into the nodes array for the node n of element i, for now n=0,1,2 for triangles. Thus element 0 will have nodes (elements(i,0),elements(i,1),...)

41.60 project

int
project(realArray & x, MappingProjectionParameters & mpParameters)

Description: Project points onto the surface

x (input) : project these points.

mpParameters : holds auxillary data to aid in the projection.

41.61 buildFromAMapping

intArray
buildFromAMapping(Mapping & map, intArray &maskin = nullIntArray())

Description: Builds an unstructured mapping from another mapping. There are no duplicate nodes. Degenerate elements occurring from coordinate singularities and periodic boundaries are detected and the appropriate element (hex, prism, pyramid, tet) is created in the UnstructuredMapping. For example, a spherical polar mesh will, in general, have all four element types with pyramids at the spherical singularity, tetrahedron connecting the pyramids to the polar axes, prisms along each polar axis and hexahedra everywhere else. A mask array can optionally be provided to exclude vertices/elements from the new UnstructuredMapping. However, building a new UnstructuredMapping from a masked UnstructuredMapping is NOT yet supported. The implementor is a bit lazy.

map (input) : Mapping to use.

maskin (input) : pointer to a vertex mask array to determine which nodes/elements to use

Returns : An IntegerArray mapping the vertices in the original Mapping to the vertices in the new UnstructuredMapping. If the value of the returned array is -1 at any vertex, then that vertex was masked out of the original mapping.

Comments : Currently the code implements a rather complex algorithm to assign vertex id's to the boundary nodes. The complexity of the coding is due to the possibility of polar singularities (with the possible occurance of a spherical singularity) as well as periodic boundaries. These special cases can occur on any side of any coordinate axis in 2 and 3d. The approach became more complicated than originally intended, there may be a more straightforward way and any suggestions are welcome.

41.62 get from an ascii file

int
get(const String & fileName)

Description: Read the unstructured grid from an ascii file.

fileName (input) : name of the file to save the results in.

41.63 put to an ascii file

int
put(const String & fileName) const

Description: Save the unstructured grid to an ascii file.

fileName (input) : name of the file to save the results in.

41.64 Constructor

void
buildFromACompositeGrid(CompositeGrid &cg)

Description: build an unstructured mapping from a composite grid

cg (input) : a composite grid that may or may not be a hybrid grid

Comments : The composite grid has no restrictions, it could be an overlapping grid or hybrid mesh. In the case of an overlapping grid, the UnstructuredMapping essentially consists of overlapping sections and holes that have no connectivity information. A hybrid mesh becomes one consistent UnstructuredMapping.

41.65 getColour

aString
getColour( ) const

Purpose: Get the colour of the grid.

Return value : the name of the colour.

41.66 setColour

int
setColour(const aString & colour)

Purpose: Set the colour for the grid.

colour (input) : the name of the colour such as ”red”, ”green”,...

41.67 eraseUnstructuredMapping

void
eraseUnstructuredMapping(GenericGraphicsInterface &gi)

Description: purge all display lists for the unstructured mapping

41.68 getColour

aString
getColour( ) const

Purpose: Get the colour of the grid.

Return value : the name of the colour.

41.69 setColour

int
setColour(const aString & colour)

Purpose: Set the colour for the grid.

colour (input) : the name of the colour such as ”red”, ”green”,...

41.70 eraseUnstructuredMapping

void
eraseUnstructuredMapping(GenericGraphicsInterface &gi)

Description: purge all display lists for the unstructured mapping

41.71 addTag

EntityTag &
addTag(const EntityTypeEnum entityType, const int entityIndex, const std::string tagName,
const void *tagData, const bool copyTag, const int tagSize)

Purpose: add an EntityTag to a specific entity in the mesh

entityType (input) : the EntityTypeEnum of the entity

entityIndex (input): the index of the entity

tagName (input): name to give the tag instance

tagData (input): data stored by the tag

copyTag (input): deep copy tagData if copyTag==true, shallow copy if false

tagSize (input): if copyTag==true, this is the size of the tagData

Returns : a reference to the added EntityTag

41.72 deleteTag

int
deleteTag(const EntityTypeEnum entityType, const int entityIndex,
const EntityTag &tagToDelete)

Purpose: delete an EntityTag from the mesh

entityType (input) : the EntityTypeEnum of the entity

entityIndex (input): the index of the entity

tagToDelete (input): a reference to a tag specifying the deletion

Returns : 0 if successfull

41.73 deleteTag

int
deleteTag(const EntityTypeEnum entityType, const int entityIndex,
const std::string tagToDelete)

Purpose: delete an EntityTag from the mesh

entityType (input) : the EntityTypeEnum of the entity

entityIndex (input): the index of the entity

tagToDelete (input): a string specifying the name of the tag to delete

Returns : 0 if successfull

41.74 hasTag

bool
hasTag(const EntityTypeEnum entityType, const int entityIndex, const std::string tag)

Purpose: check to see if an entity has a particular tag

entityType (input) : the EntityTypeEnum of the entity

entityIndex (input): the index of the entity

tag (input): a string specifying the name of the tag in question

Returns : true if the tag exists on the entity

41.75 getTag

EntityTag &
getTag(const EntityTypeEnum entityType,
const int entityIndex, const std::string tagName)

Purpose: obtain a reference to a tag on a specific entity

entityType (input) : the EntityTypeEnum of the entity

entityIndex (input): the index of the entity

tagName (input): a string specifying the name of the tag in question

Returns : the tag requested

Throws : TagError if the tag is not found

41.76 getTagData

void *
**getTagData(const EntityTypeEnum entityType, const int entityIndex,
const std::string tag)**

Purpose: obtain the the data in a tag

entityType (input) : the EntityTypeEnum of the entity

entityIndex (input): the index of the entity

tag (input): a string specifying the name of the tag in question

Returns : NULL if the tag did not exist

41.77 setTagData

int
**setTagData(const EntityTypeEnum entityType, const int entityIndex,
const std::string tagName,
const void *data, const bool copyData, const int tagSize)**

Purpose: set the data in an existing tag

entityType (input) : the EntityTypeEnum of the entity

entityIndex (input): the index of the entity

tagName (input): a string specifying the name of the tag in question

data (input): data stored by the tag

copyTag (input): deep copy tagData if copyTag==true, shallow copy if false

tagSize (input): if copyTag==true, this is the size of the tagData

Returns : 0 if successfull

41.78 maintainTagToEntityMap

void
maintainTagToEntityMap(bool v)

Purpose: turn on/off maintainance of the mapping from tags to thier entities

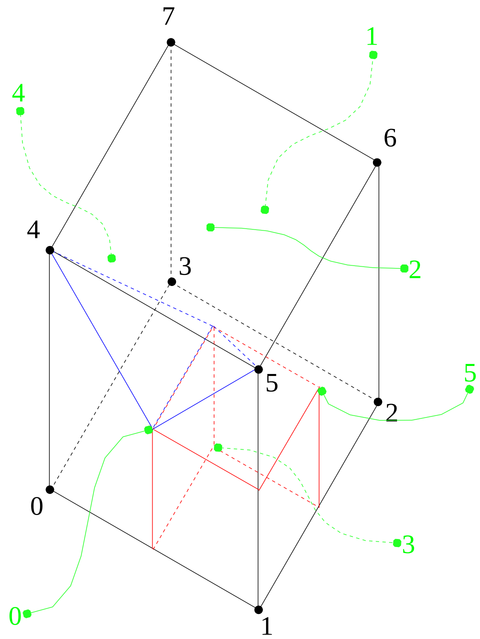
v (input) : if true turn on the tag to entity mapping, if false turn it off

Note: If v==true, this method will build the mapping. If false, it will destroy the mapping

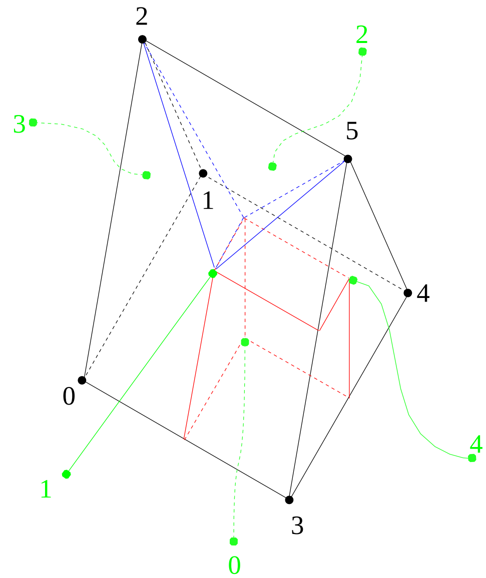
41.79 maintainsTagToEntityMap

bool
maintainsTagToEntityMap( ) const

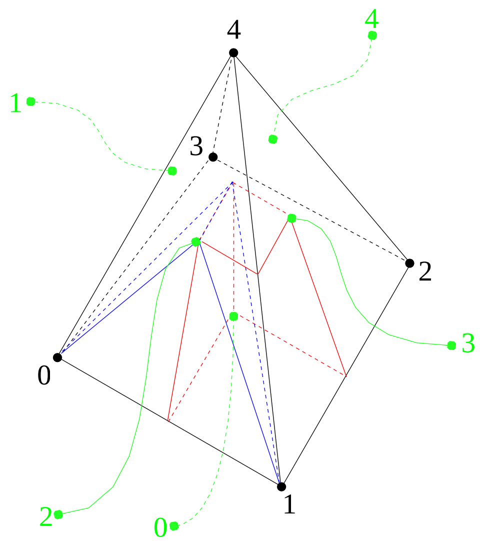
Purpose: return true if the Mapping maintains the list of entities with a given tag == / compare the vertices of an entity to a list of vertices, return true if the list specifies the entity // FALSE : no entities of this type created yet! // FALSE : invalid entity id given! // FALSE : the number of vertices do not match in each entity // FALSE : number of vertices do not match! // two entities are the same if thier vertices are the same, note that the ordering can // be reversed. // first find the starting point for each entity // the starting point is the lowest vertex id // FALSE : minimum vertex index does not match // now check the vertices in the current order // now check in the opposite direction (only the previous did not work!) / setAsGhost takes an entity and adjusts the data structures to make it a ghost // if the entity mask array is there (if not build it?) set the mask // now add the info as a tag // note this is a simple tag; the only data is the index ”entity” // later we may allow construction using connectivity info // vertices do have an ”orientation” relative to thier edges, the lowest vertex index is +ive // later we may allow construction using connectivity info / connectivityBuilder directs the construction of the connectivity arrays, it returns true if successfull if (!entities[to]) return false; // we don’t have enough information // there is no downward from here! // XXX else add generic downward builder here! /// we always have this if there are Regions // XXX else add generic downward builder here! // XXX else add generic downward builder here! / specifyConnectivity tells the mapping to use the given connectivity information rather than building it / delete specific connectivity information / delete all connectivity information for a specific entity type / delete ALL the connectivity information



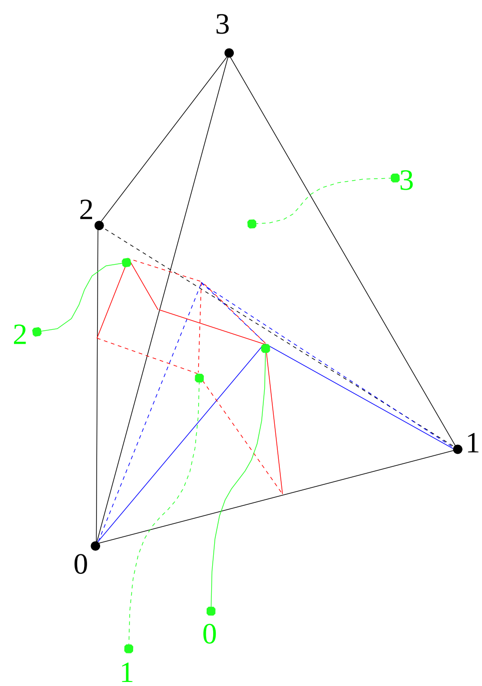
(a)



(b)



(c)



(d)

Figure 24: Canonical orderings for the 3D finite element zoo: (a) hexahedra; (b) triangle prisms; (c) pyramids; (d) tetrahedra. Black indicates vertex numbers, green indicates face indices. Smple corners are drawn in red and sides are in blue.

42 Class Fraction

This class is used to define “fractions”, the ratio of two integers. Fractions can represent infinity and -infinity with a zero numerator and nonzero denominator. Thus $1/0$ is infinity and $-1/0$ is -infinity. We define $2/0$ be be greater than $1/0$.

42.1 Constructors

`Fraction( int n, int d=1 )`	define a fraction, `n` = numerator, `d` = denominator

Note that we do not know how to construct a fraction from a real number.

42.2 Member Functions

The relational operators $\le$, $<$, $\ge$, $>$ and $==$ are defined for the comparison of two fractions or a fraction and a real number. In addition, the arithmetic operators $+$, $-$, $*$ and $/$ are defined for two objects of type Fraction (or a Fraction and a real or int). NOTE: By definition the result of the operators $+$, $-$, $*$, or $/$ between a Fraction and a real results in a real.
Here are the member functions that can be used to access the numerator and denominator

`int setNumerator()`	set the numerator
`int setDenominator()`	set the denominator
`int getNumerator()`	get the numerator
`int getDenominator()`	get the denominator

43 Class Bound

A bound is defined as a real number, a fraction or null. The bound class implements the bound and supplies functions for comparing bounds. Bounds allow rational numbers to be specified precisely. Bounds can represent infinity and -infinity by fractions with a zero numerator and nonzero denominator. Thus $1/0$ is infinity and $-1/0$ is -infinity. We define $2/0$ be be greater than $1/0$.

43.1 enum types

```
enum boundType{ realNumber, fraction, null };
```

43.2 Constructors

`Bound()`	default constructor, boundType=null
`Bound( real x0 )`	define a bound from a real number
`Bound( int i )`	define a bound from a int
`Bound( Fraction f0 )`	define a bound from a fraction

43.3 Member Functions

The relational operators $\le$, $<$, $\ge$, $>$ and $==$ are defined for the comparison of two bounds or a bound and a real number, or a bound and a fraction. In addition the arithmetic operators $+$, $-$, $*$ and $/$ are defined for two objects of type Bound. There are also member functions to assign and retrieve values

`void set( real value )`	assign a real value to the bound
`void set( int value )`	assign an integer value to the bound
`void set( int n, int d )`	assign a numerator and denominator
`void get( boundType bt, real x, Fraction f )`	get boundType and value

44 Class Triangle

The `Triangle` class is used to represent a triangle in three dimensional space. It has a function for determining how two triangles intersect that is used by the `IntersectionMapping` class.

44.1 Constructor

Triangle()

Purpose: Default Constructor, make a default triangle with vertices (0,0,0), (1,0,0), (0,1,0)

44.2 Constructor(const real x1[],x2[],x3[])

Triangle(const real x1_[3], const real x2_[3], const real x3_[3])

Purpose: Create a triangle with vertices x1,x2,x3

x1,x2,x3 (input) : the three vertices of the triangle

44.3 Constructor(const RealArray & x1,x2,x3)

Triangle(const RealArray & x1_, const RealArray & x2_, const RealArray & x3_)

Purpose: Create a triangle with vertices x1,x2,x3

x1,x2,x3 (input) : the three vertices of the triangle

44.4 Constructor(grid)

Triangle(const realArray & grid,
const int & i1,
const int & i2,
const int & i3,
const int & choice =0,
const int & axis =axis1)

Purpose: Build a triangle from a quadrilateral on the face of a grid grid, This constructor just calls the corresponding `setVertices` function. See the comments there.

44.5 setVertices(const real x1,x2,x3)

void
setVertices(const real x1_[3], const real x2_[3], const real x3_[3])

Purpose: Assign the vertices to a triangle.

x1,x2,x3 (input) : the three vertices of the triangle

44.6 setVertices(const RealArray & x1,x2,x3)

void
setVertices(const RealArray & x1_, const RealArray & x2_, const RealArray & x3_)

Purpose: Assign the vertices to a triangle.

x1,x2,x3 (input) : the three vertices of the triangle

44.7 setVertices

void
setVertices(const realArray & grid,
const int & i1,
const int & i2,
const int & i3,
const int & choice =0,
const int & axis =axis3)

Purpose: Form a triangle from a quadrilateral on the face of a grid grid, there are six possible choices.

grid (input) : and array containing the four points `grid(i1+m,i2+n,i3,0:2)`, `m=0,1`, `n=0,1`.

i1,i2,i3 (input) : indicates which quadrilateral to use

choice, axis (input) : These define which of 6 poissible triangles to choose:

choice=0, axis=axis3(==2) : use points (i1,i2,i3), (i1+1,i2,i3), (i1,i2+1,i3). Lower left triangle in the plane i3==constant.

choice=1, axis=axis3(==2) : use points (i1+1,i2+1,i3), (i1,i2+1,i3), (i1+1,i2,i3). Upper right triangle in the plane i3==constant.

choice=0, axis=axis2(==1) : use points (i1,i2,i3), (i1,i2,i3+1), (i1+1,i2,i3).

choice=1, axis=axis2(==1) : use points (i1+1,i2,i3+1), (i1+1,i2,i3), (i1,i2,i3+1).

choice=0, axis=axis1(==0) : use points (i1,i2,i3), (i1,i2+1,i3), (i1,i2,i3+1).

choice=1, axis=axis1(==0) : use points (i1,i2+1,i3+1), (i1,i2,i3+1), (i1,i2+1,i3).

The figure below shows the two choices for axis=axis3:

```
      x2
   x3 ----------- x1
      |\         |
      |  \   1   |
      |    \     |
      | 0    \   |
      |________\|x3
     x1        x2
```

44.8 area

real
area() const

Purpose: return the area of the triangle

44.9 display

void
display(const aString & label =blankString) const

Purpose: print out the vertices and the normal.

44.10 tetraheadralVolume

double
tetraheadralVolume(const real a[], const real b[], const real c[], const real d[]) const

Purpose: Return the approximate volume (actually 6 times the volume) of the tretrahedra formed by the points (a,b,c,d)

44.11 intersects

bool
intersects(Triangle & tri, real xi1[3], real xi2[3]) const

Purpose: Determine if this triangle intersect another.

tri (input) : intersect with this triangle.

xi1, xi2 (output) : if the return value is true then these are the endpoints of the line of intersection between the two triangles.

Return value : TRUE if the triangles intersect, false otherwise.

44.12 intersects

bool
intersects(Triangle & triangle, RealArray & xi1, RealArray & xi2) const

Purpose: Determine if this triangle intersect another.

tri (input) : intersect with this triangle.

xi1, xi2 (output) : if the return vaule is true then these are the endpoints of the line of intersection between the two triangles.

Return value : TRUE if the triangles intersect, false otherwise.

44.13 intersects

bool
intersects(real x[3], real xi[3]) const

Purpose: Determine if this triangle intersects a ray starting at the point x[] and extending to y=+infinity.

x (input) : find the intersection with a vertical ray starting at this point.

xi (output) : if the return value is true then this is the intersection point.

Return value : TRUE if the ray intersects the triangle, false otherwise.

44.14 intersects

bool
intersects(RealArray & x, RealArray & xi) const

Purpose: Determine if this triangle intersects a line starting at the point x and extending to y=+infinity.

x (input) : find the intersection with a vertical ray starting at this point.

xi (output) : if the return value is true then this is the intersection point.

Return value : TRUE if the ray intersects the triangle, false otherwise.

44.15 getRelativeCoordinates

int
getRelativeCoordinates(const real x[3],
real & alpha1,
real & alpha2,
const bool & shouldBeInside =TRUE) const

Description: Determine the coordinates of the point x with respect to this triangle. I.e. solve for alpha1,alpha2 where x-x1 = alpha1 * v1 + alpha2 * v2

where v1=x2-x1 and v2=x3-x1 are two vectors from the sides of the triangle, (x1,x2,x3) Solve

```
        [ v1.v1 v1.v2 ] [ alpha1 ] = [ v1.x ]
        [ v1.v2 v2.v2 ] [ alpha2 ] = [ v2.x ]
  alpha1 = ( v1.x * v2.v2 - v2.x * v1.v2 ) /( v1.v1 * v2.v2 - v1.v2 * v1.v2 )
  alpha2 = ( v1.x * v2.v2 - v2.x * v1.v2 ) /( v1.v1 * v2.v2 - v1.v2 * v1.v2 )
```

x (input) : find coordinates of this point.

alpha1, alpha2 (output) : relative coordinates.

shouldBeInside (input) : if true, this routine will print out a message if alpha1 or alpha are not in the range [0,1] (+/- epsilon), AND return a value of 1

Return value : 0 on sucess, 1 if shouldBeInside==TRUE and the point is not inside.

References

[1] W. HENSHAW, *The overture hyperbolic grid generator, user guide, version 1.0*, Research Report UCRL-MA-??, Lawrence Livermore National Laboratory, 1999.

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